CACAO seminar – March 2009

Hardware Operators for Pairing-Based Cryptography

— Part II: Because speed also matters —

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Outline of the talk

- Previously in the Jean-Luc Beuchat Tour
- A closer look at the algorithm
- Accelerating the η_T pairing
- Accelerating the final exponentiation
- Implementation results
- Concluding thoughts

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Bilinear pairings

- $\mathbb{G}_1 = \langle P \rangle$: additively-written cyclic group of prime order $\#\mathbb{G}_1 = \ell$
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- A bilinear pairing on $(\mathbb{G}_1, \mathbb{G}_2)$ is a map

$$\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$$

  that satisfies the following conditions:

  - non-degeneracy: $\hat{e}(P, P) \neq 1_{\mathbb{G}_2}$ (equivalently $\hat{e}(P, P)$ generates $\mathbb{G}_2$)
  - bilinearity:

    $$\hat{e}(Q_1 + Q_2, R) = \hat{e}(Q_1, R) \cdot \hat{e}(Q_2, R) \qquad \hat{e}(Q, R_1 + R_2) = \hat{e}(Q, R_1) \cdot \hat{e}(Q, R_2)$$

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- Immediate property: for any two integers k_1 and k_2

$$\hat{e}(k_1 P, k_2 P) = \hat{e}(k_2 P, k_1 P) = \hat{e}(P, P)^{k_1 k_2}$$

Pairings in cryptography

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- One-round three-party key agreement (Joux, 2000)
- Identity-based encryption
  - Boneh-Franklin, 2001
  - Sakai-Kasahara, 2001
- Short digital signatures
  - Boneh-Lynn-Shacham, 2001
  - Zang-Safavi-Naini-Susilo, 2004
- ...

The Tate pairing over supersingular elliptic curves

- We first define
  - $\mathbb{F}_{p^m}$, a finite field, with $p = 2$ or 3
  - E, a supersingular elliptic curve defined over $\mathbb{F}_{p^m}$
  - ℓ, a large prime factor of $\#E(\mathbb{F}_{p^m})$

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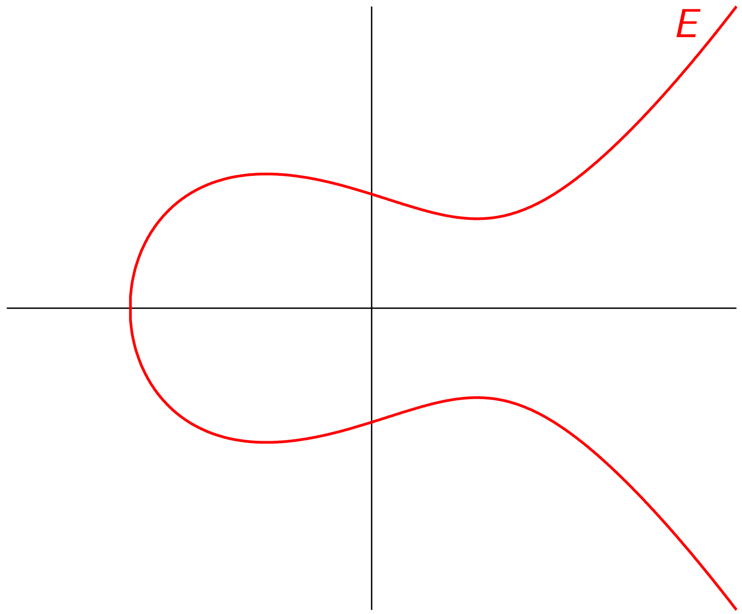
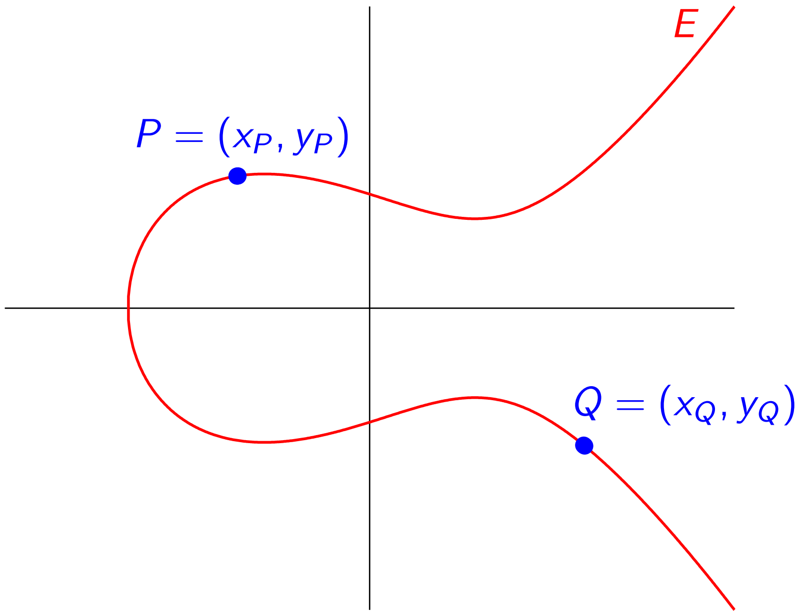
- k is the embedding degree, the smallest integer such that $\mu_\ell \subseteq \mathbb{F}_{p^{km}}^\times$
  - $k = 4$ in characteristic $p = 2$
  - $k = 6$ in characteristic $p = 3$

The Tate pairing over supersingular elliptic curves



$\hat{e}$

The Tate pairing over supersingular elliptic curves



$$\begin{array}{cccc} \hat{e} & : & E(\mathbb{F}_{p^m})[\ell] \times E(\mathbb{F}_{p^m})[\ell] \\ & & (\quad P \quad , \quad Q \quad) \end{array}$$

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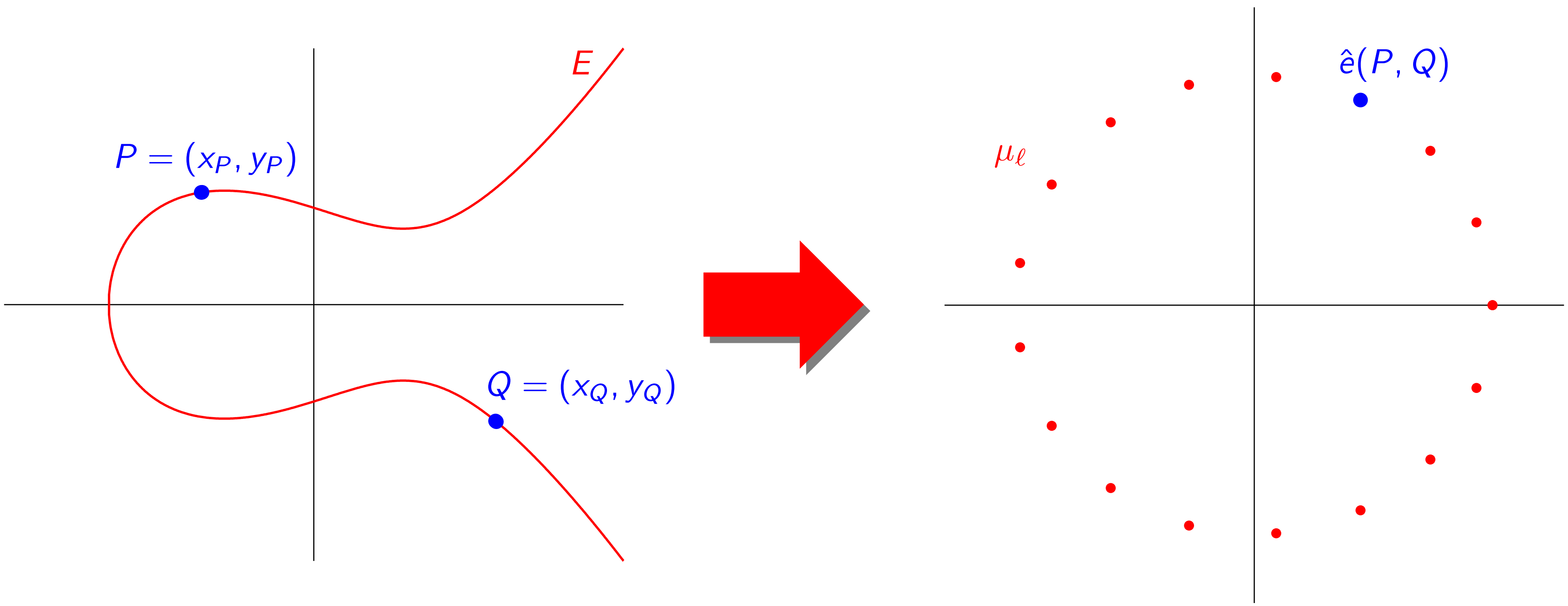

$$\begin{array}{rcccl} \hat{e} : & E(\mathbb{F}_{p^m})[\ell] \times E(\mathbb{F}_{p^m})[\ell] & \longrightarrow & \mu_\ell & \subseteq \mathbb{F}_{p^{km}}^\times \\ & (\quad P \quad , \quad Q \quad) & \longmapsto & \hat{e}(P,Q) & \end{array}$$

Security considerations

$$\hat{e} : E(\mathbb{F}_{p^m})[\ell] \times E(\mathbb{F}_{p^m})[\ell] \rightarrow \mu_\ell \subseteq \mathbb{F}_{p^{km}}^\times$$

- The discrete logarithm problem should be hard in both $\mathbb{G}_1$ and $\mathbb{G}_2$

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Base field ($\mathbb{F}_{p^m}$)	$\mathbb{F}_{2^m}$	$\mathbb{F}_{3^m}$
Lower security ($\sim 2^{64}$)	$m = 239$	$m = 97$
Medium security ($\sim 2^{80}$)	$m = 373$	$m = 163$
Higher security ($\sim 2^{128}$)	$m = 1103$	$m = 503$

- $\mathbb{F}_{2^m}$: simpler finite field arithmetic
- $\mathbb{F}_{3^m}$: smaller field extension

Computation of the Tate pairing

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  - $f(x)$, degree-m polynomial irreducible over $\mathbb{F}_p$

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  - only arithmetic over the underlying field $\mathbb{F}_{p^m}$

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- Operations over $\mathbb{F}_{p^m}$:
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  - 1 inversion

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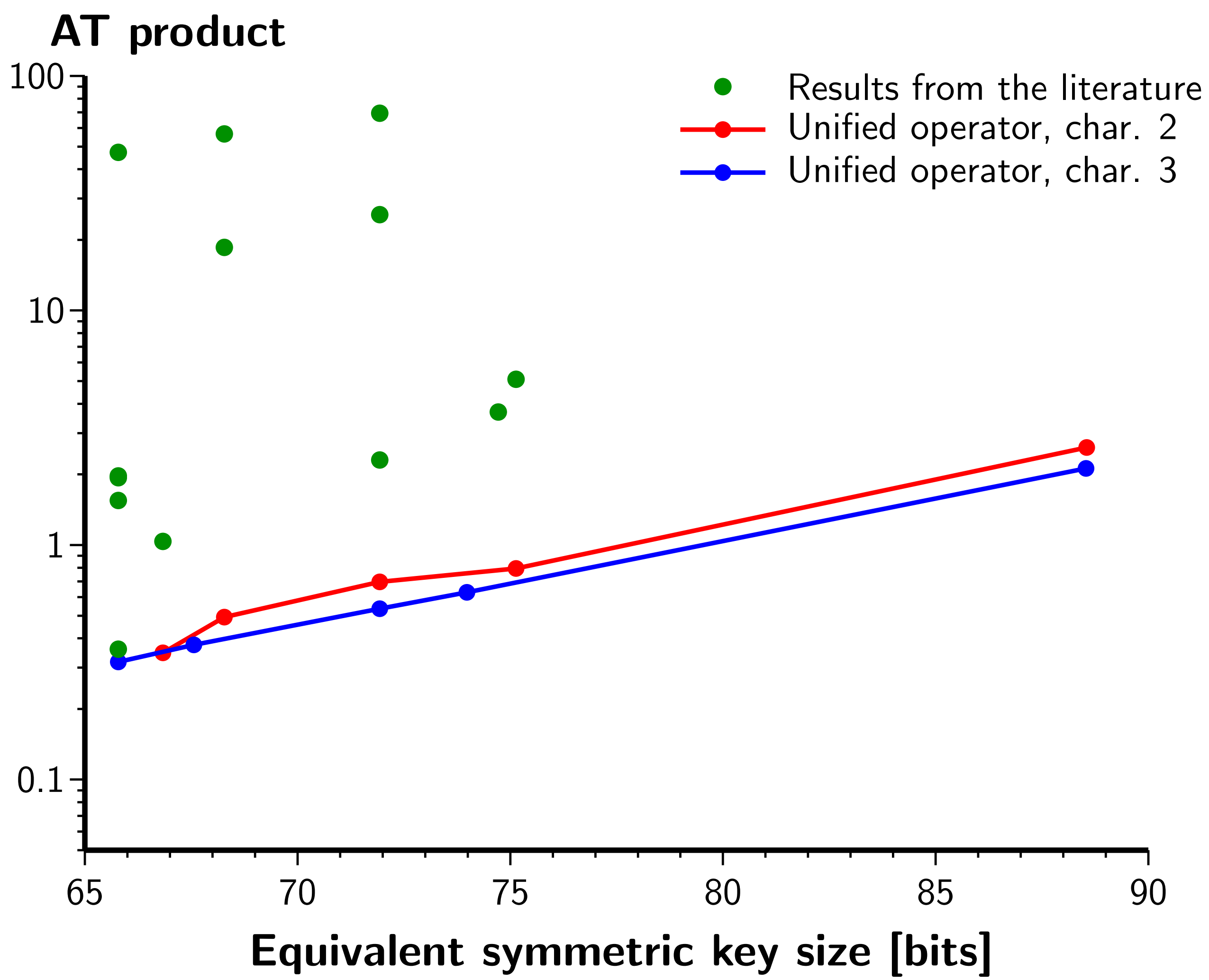
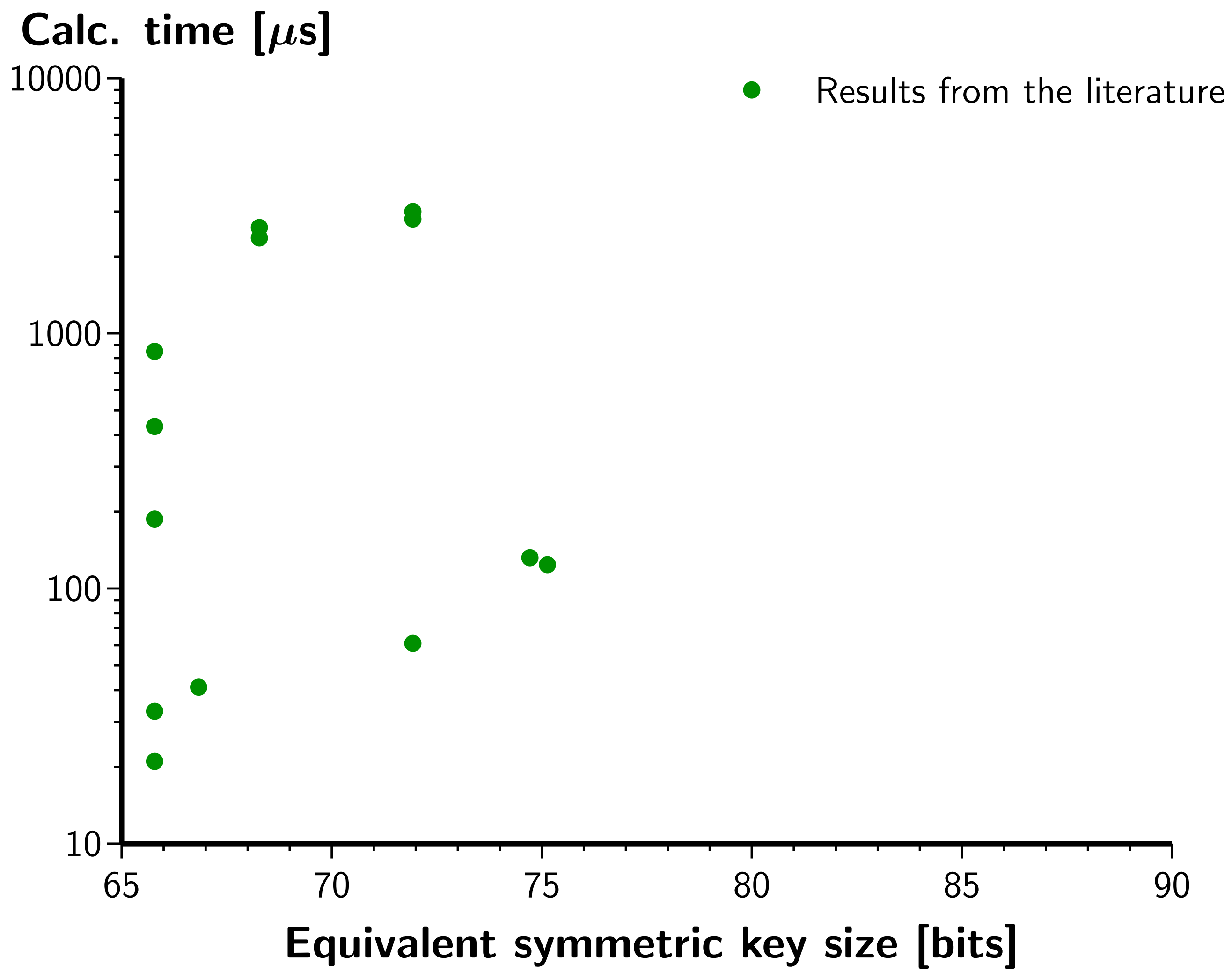
- A first idea: an all-in-one unified operator:
  - shared resources
  - scalable architecture

The best area-time product of the literature...



… But still quite slow



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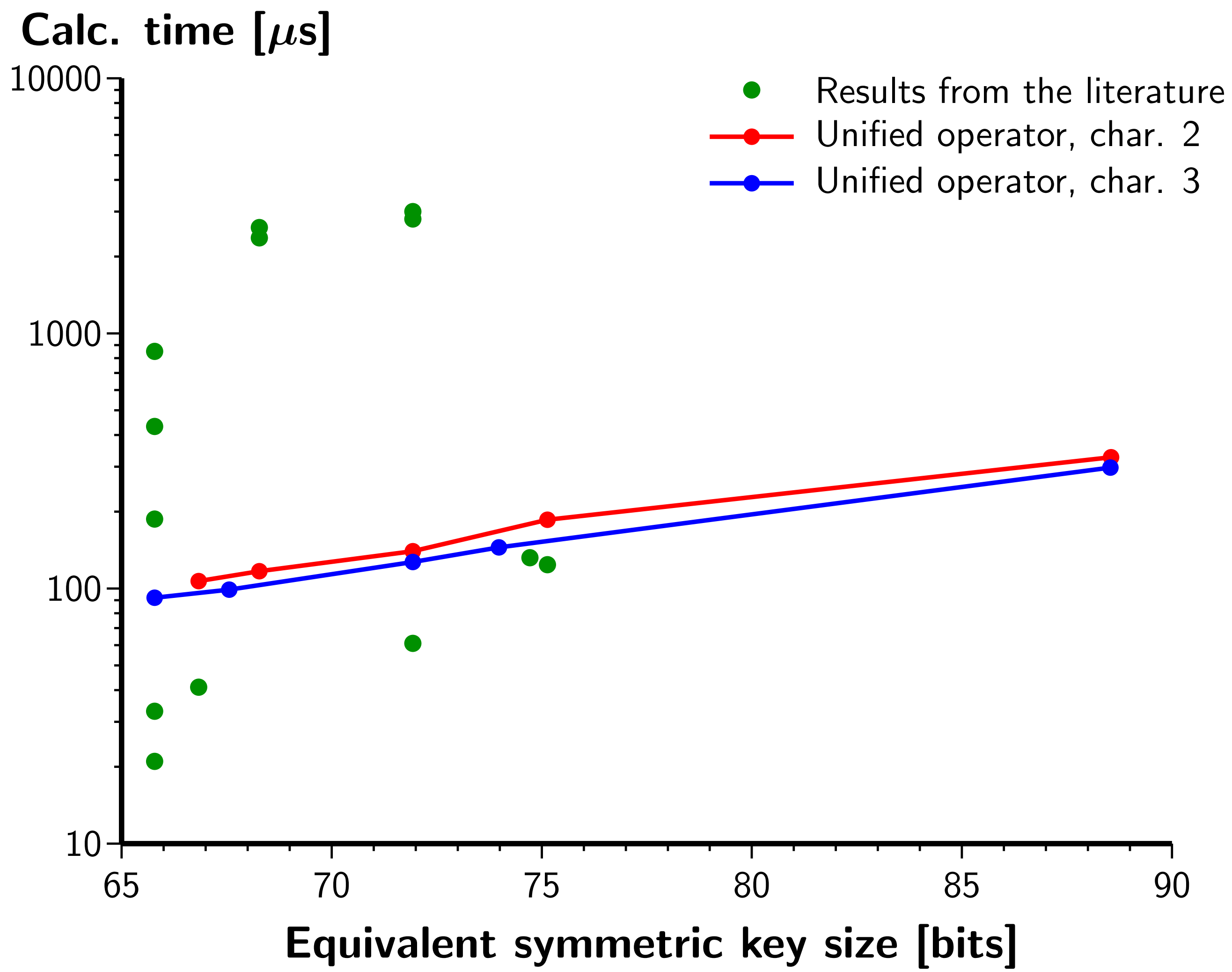
(or not the fastest, at least!)



Motivations

- High speed is more important than low resources for some cryptographic applications
- Explore the other end of the area vs. time tradeoff:
  - faster but larger than the unified operator
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- Explore the other end of the area vs. time tradeoff:
  - faster but larger than the unified operator
  - what about the area-time product?
- Accelerate the computation by extracting as much parallelism as possible...
- ... Without dramatically increasing the resource requirements

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$$\hat{e}(P, Q) = \eta_T(P, Q)$$

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  - via Miller's algorithm: loop of $(m+1)/2$ iterations
  - result only defined modulo N-th powers in $\mathbb{F}_{p^{km}}^{\times}$, with $N = \#E(\mathbb{F}_{p^m})$

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$$\hat{e}(P, Q) = \eta_T(P, Q)^M$$

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- Final exponentiation by $M = (p^{km} - 1)/N$
  - required to obtain a unique value for each congruence class
  - example in characteristic 3 ($k = 6$ and $N = 3^m + 1 \pm 3^{(m+1)/2}$):

$$M = \frac{3^{6m} - 1}{3^m + 1 \pm 3^{(m+1)/2}} = \left(3^{3m} - 1\right)\left(3^m + 1\right)\left(3^m + 1 \mp 3^{(m+1)/2}\right)$$

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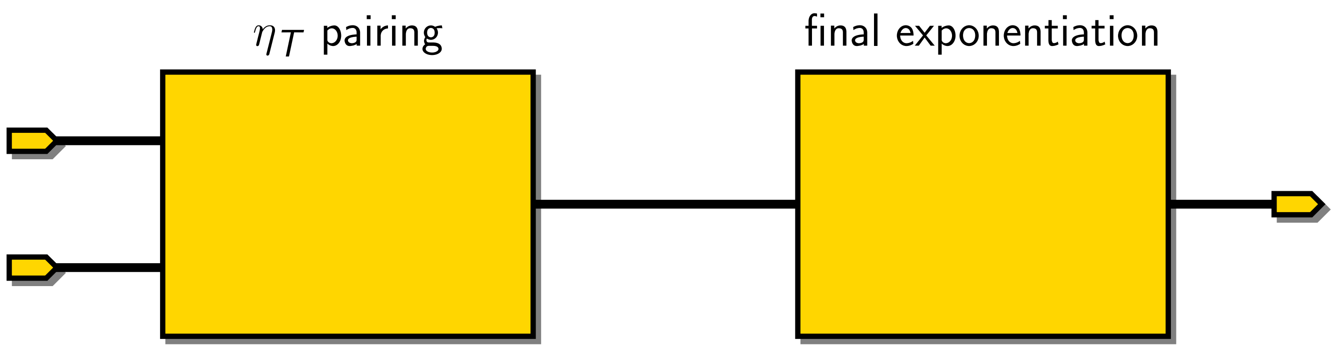
- Two distinct computational requirements $\Rightarrow$ use two distinct coprocessors

Two coprocessors for the Tate pairing

- The two operations are purely sequential



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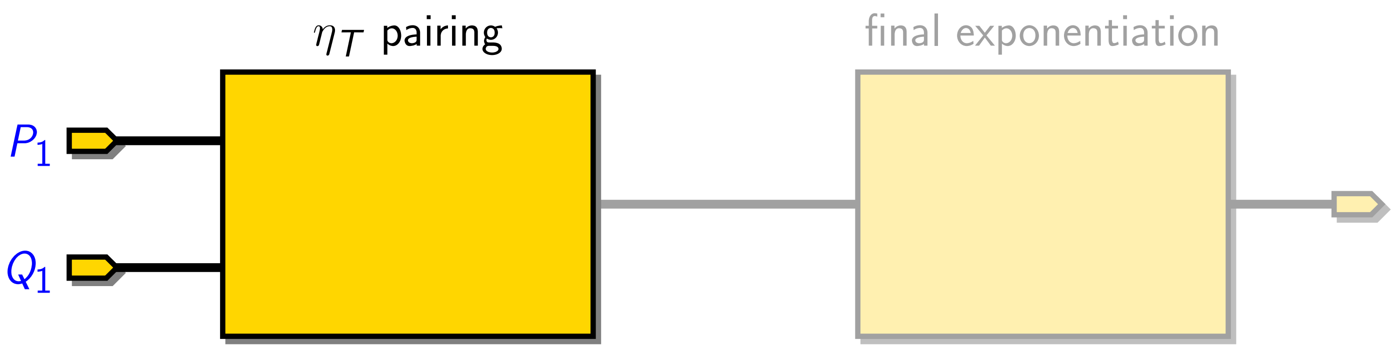

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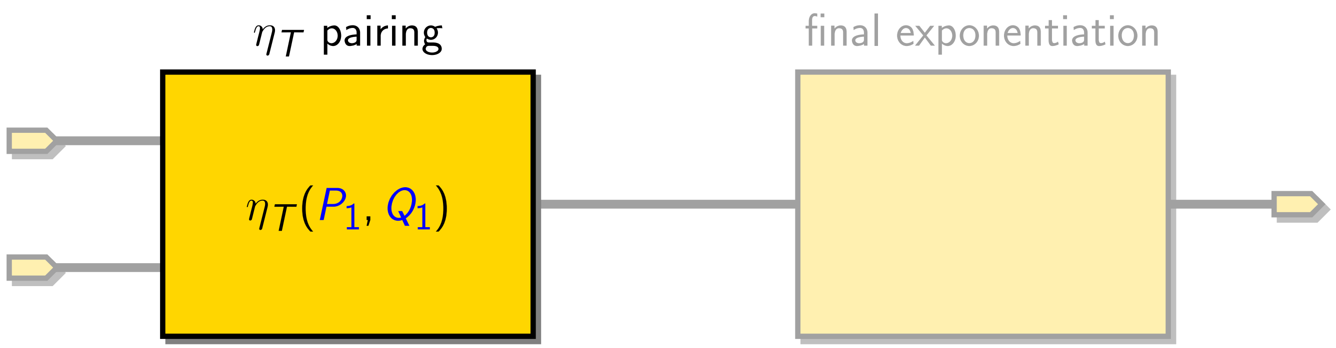

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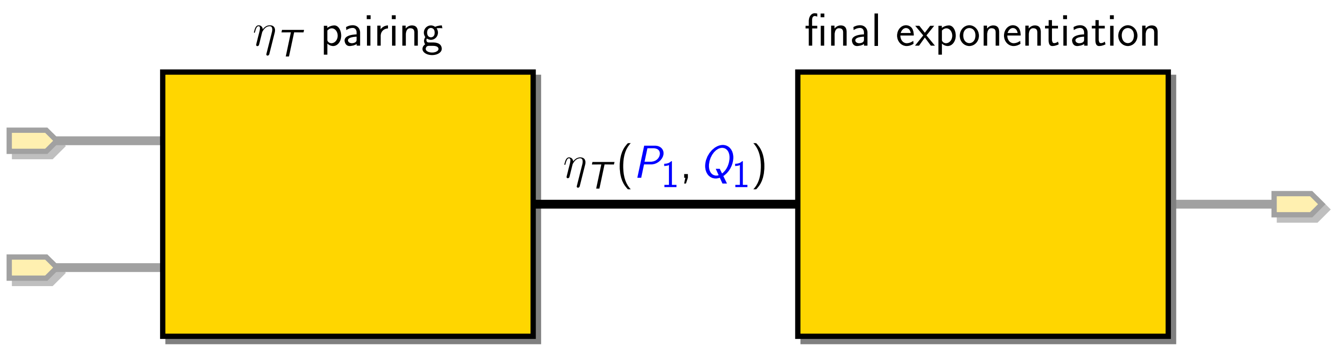

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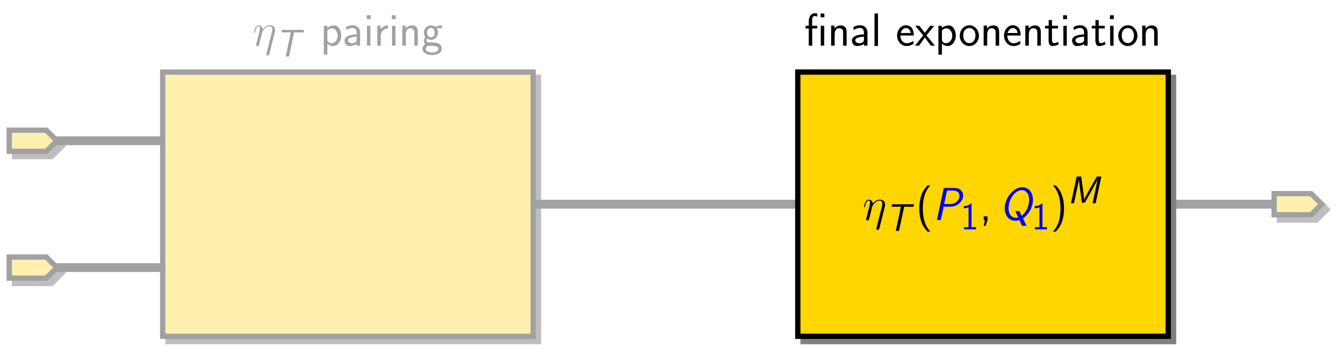

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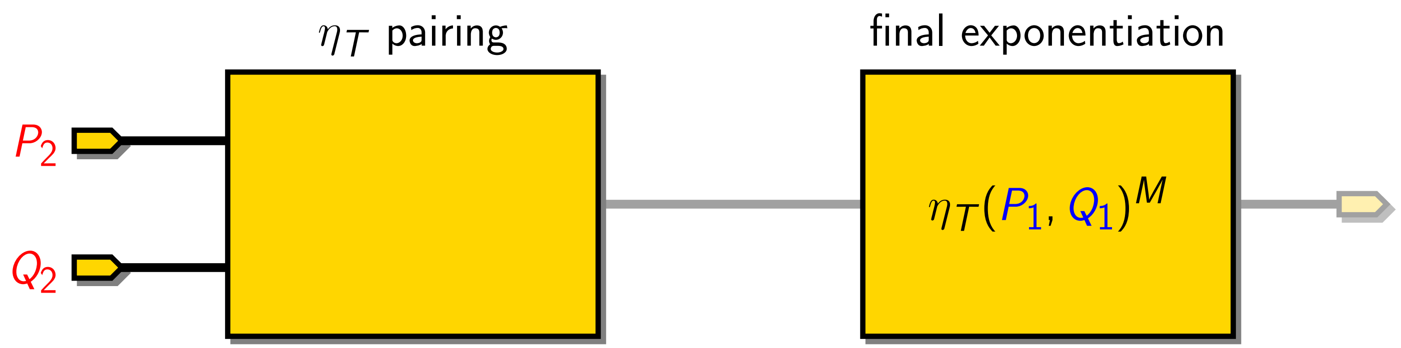

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- Pipeline the data between the two coprocessors

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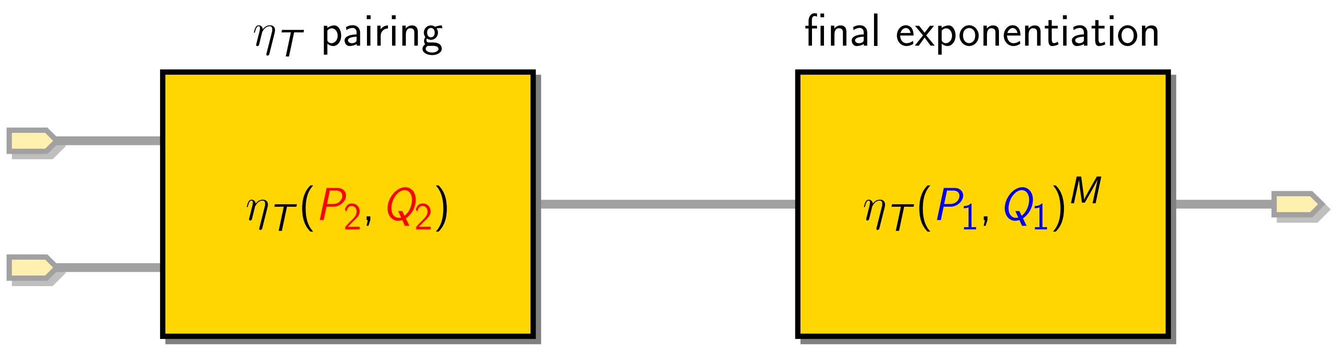

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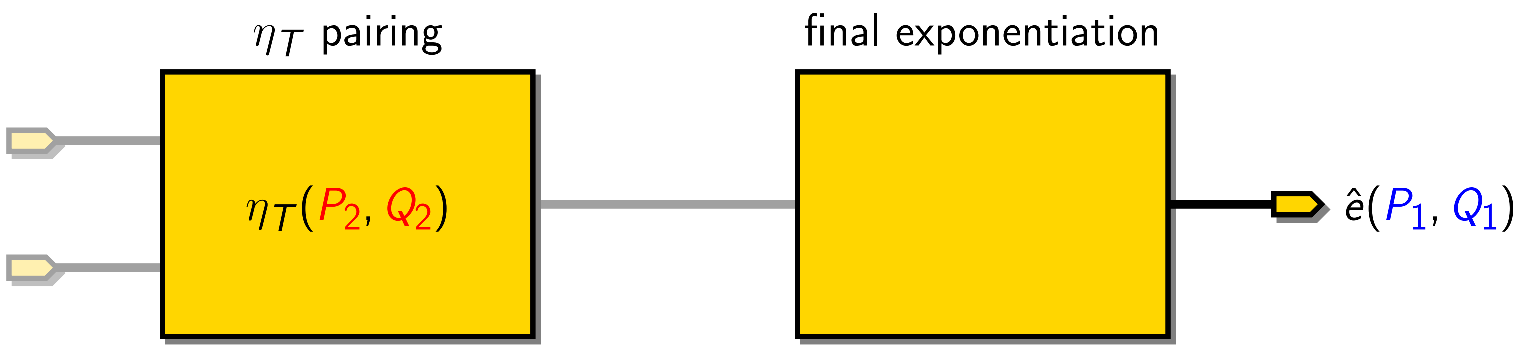

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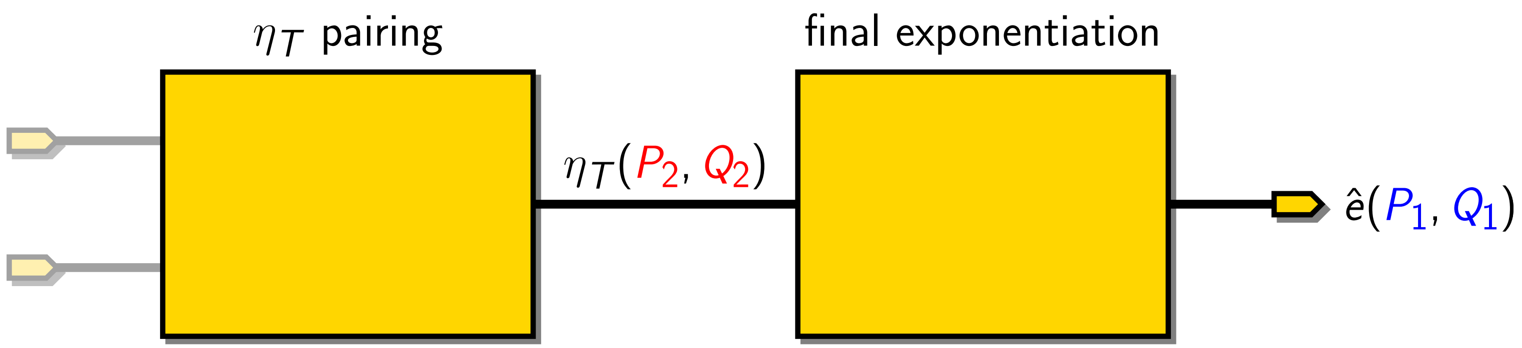

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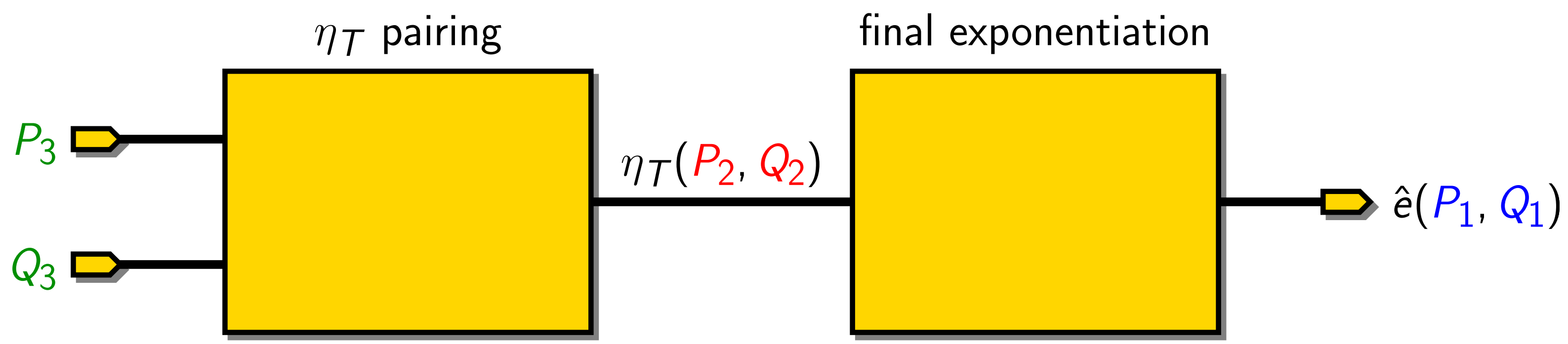

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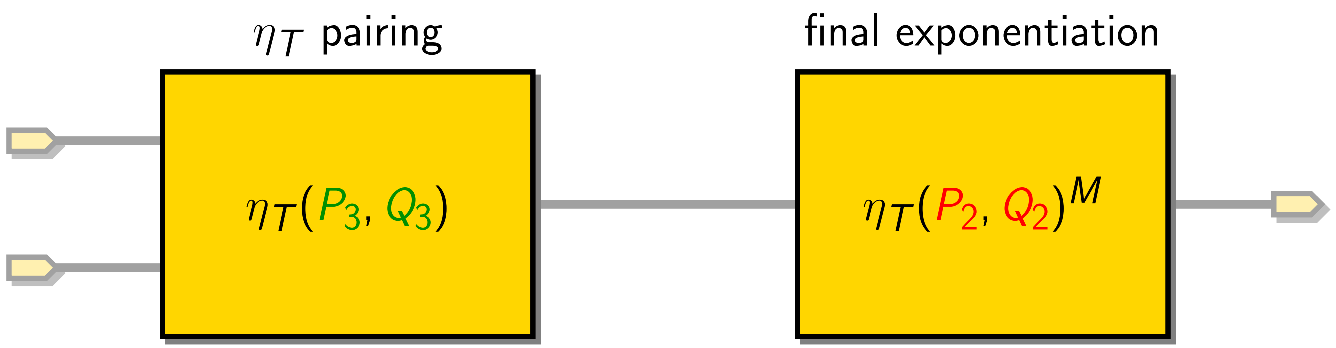

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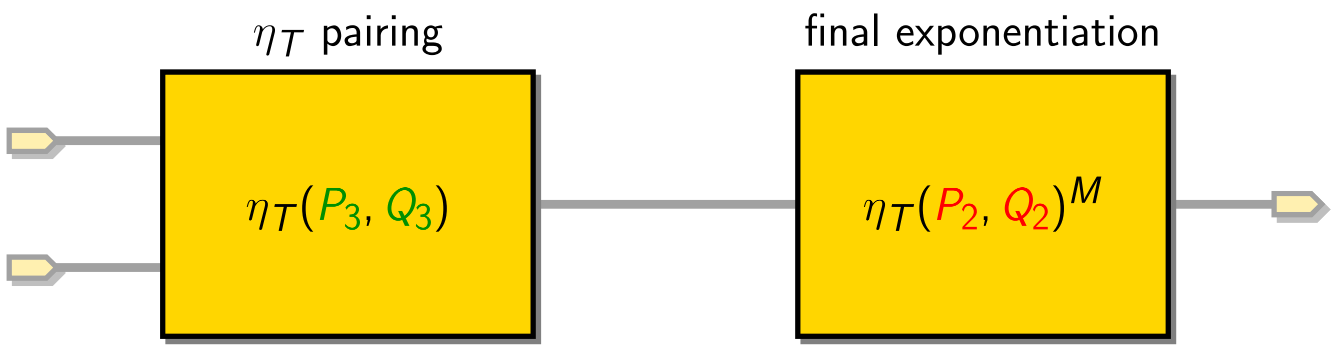

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for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

$x_Q \leftarrow x_Q^9 \pm 1 \;;\; y_Q \leftarrow -y_Q^9$

$t \leftarrow x_P + x_Q \;;\; u \leftarrow y_P y_Q$

$S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$

$R \leftarrow R \cdot S$

$R \leftarrow R^3$

end for

- Four tasks per iteration:

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  - ③ accumulate the new factor

η_T pairing algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \to \mathbb{F}^{\times}_{3^{6m}}$$

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

① $x_Q \leftarrow x_Q^9 \pm 1 \;;\; y_Q \leftarrow -y_Q^9$

② $t \leftarrow x_P + x_Q \;;\; u \leftarrow y_P y_Q$
$S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

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end for

- Four tasks per iteration:
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for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

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②	$t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$ $S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$	2 ×, 1 + $(\mathbb{F}_{3^m})$
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③	$R \leftarrow R \cdot S$	$1\times$	$(\mathbb{F}_{3^{6m}})$
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- Four tasks per iteration:
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Accelerating the η_T pairing

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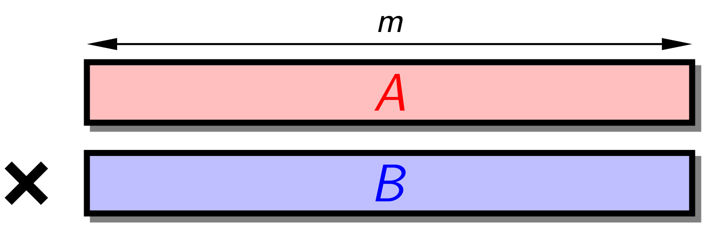

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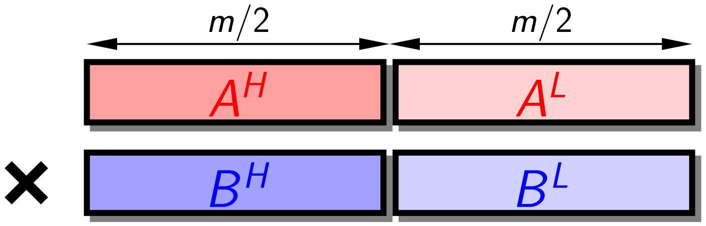

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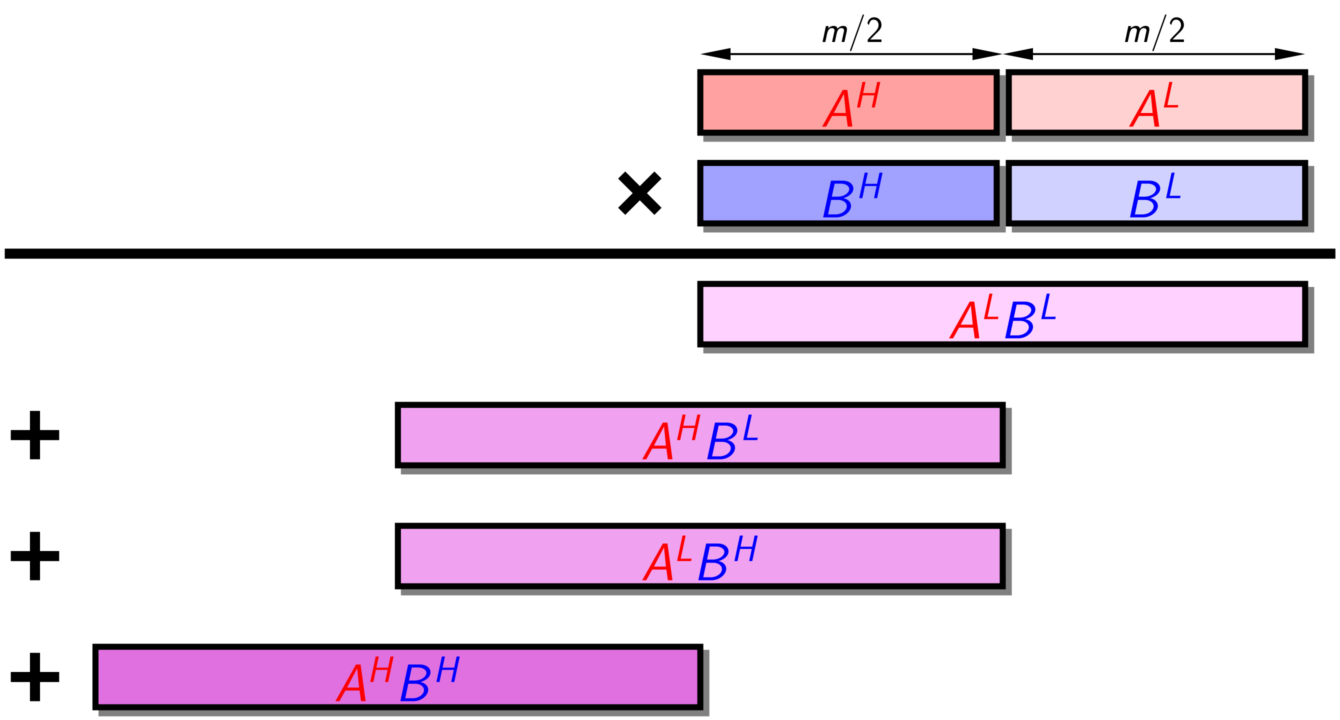

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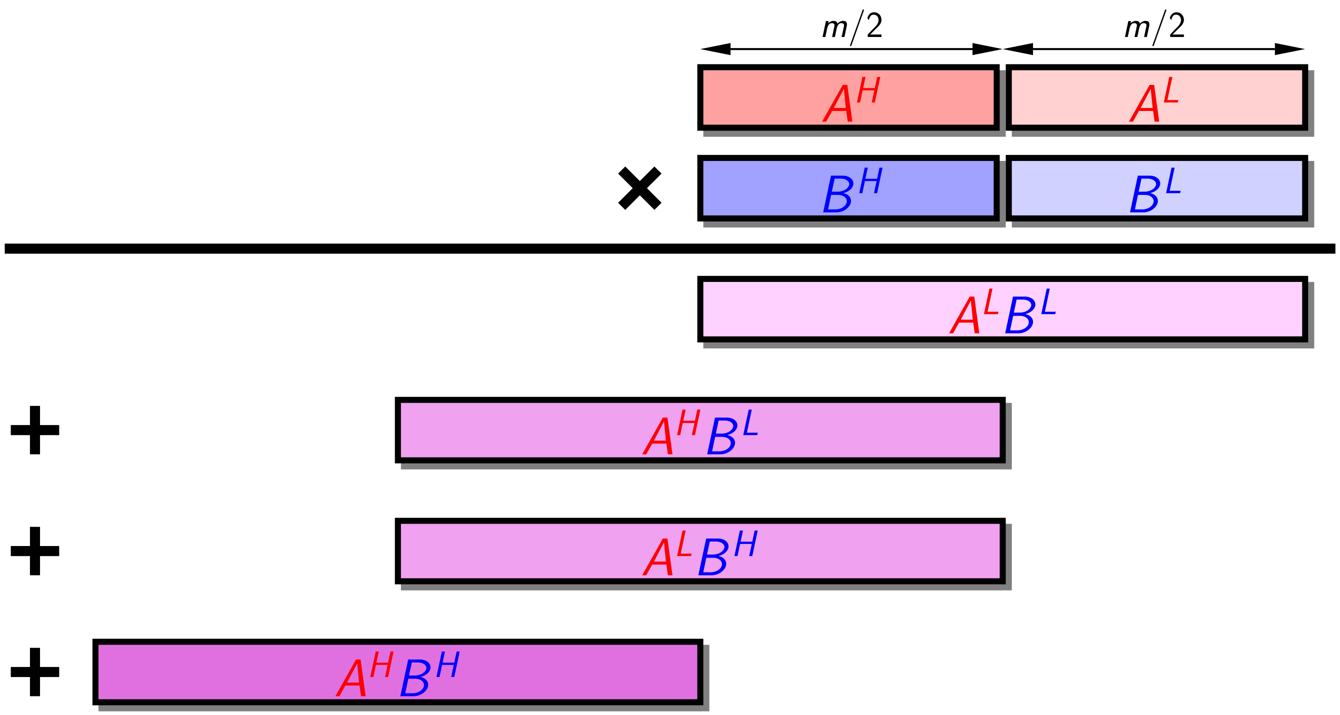

$$A^H B^L + A^L B^H = (A^H + A^L)(B^H + B^L) - A^H B^H - A^L B^L$$

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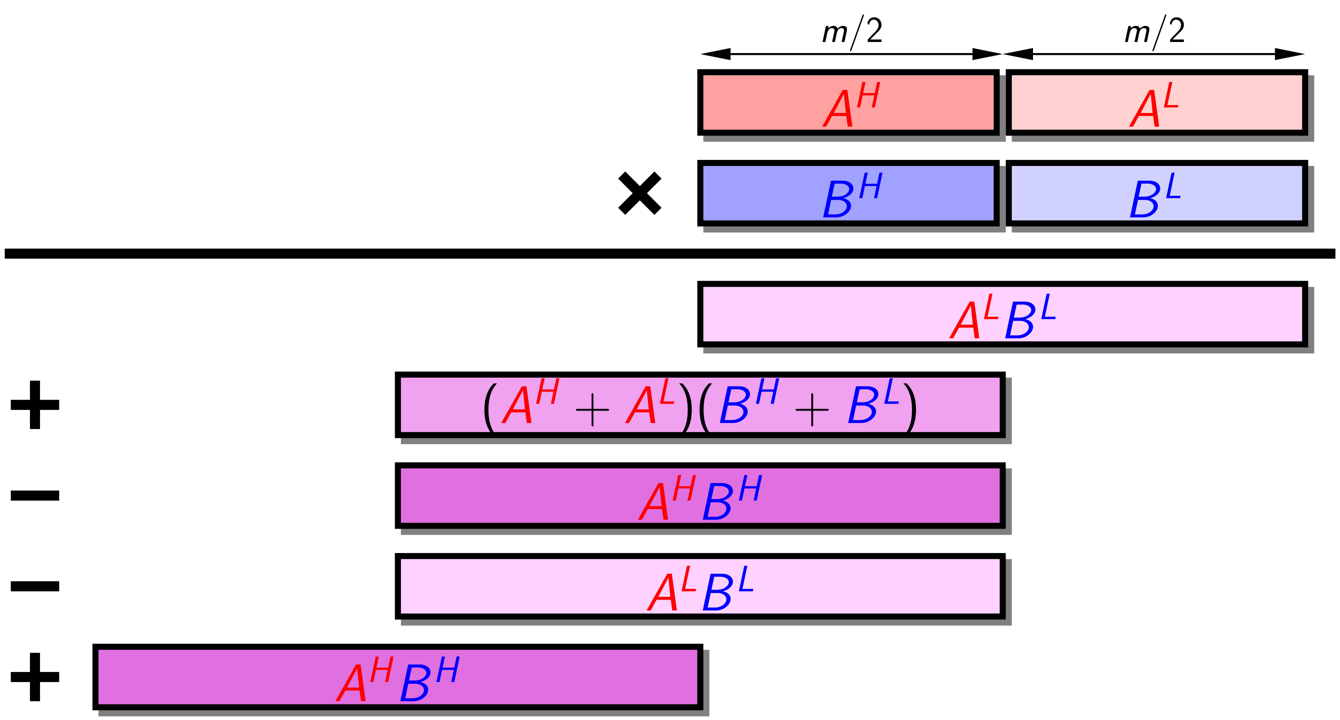

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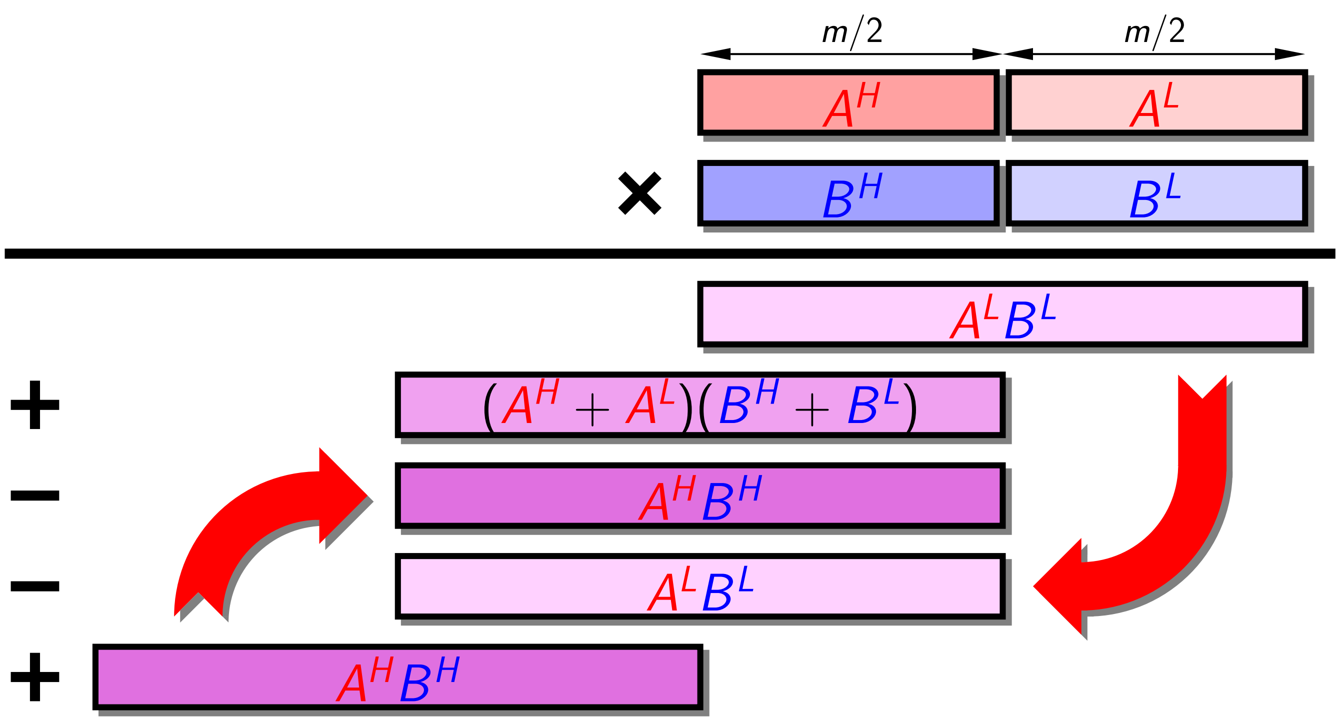

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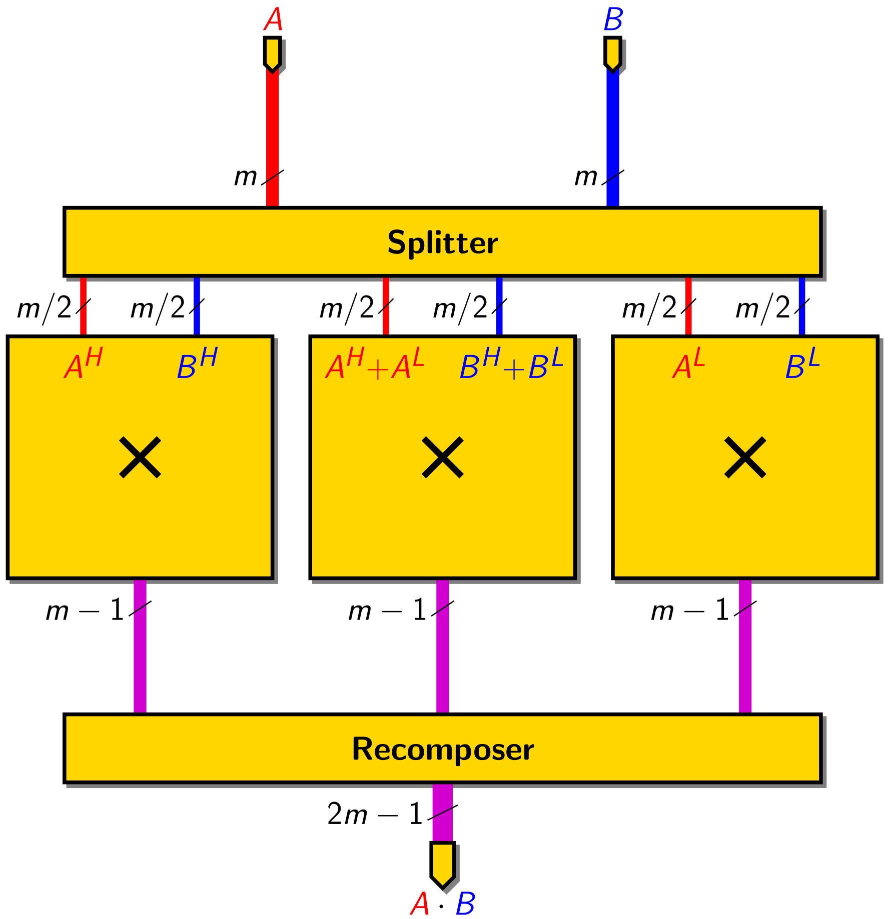
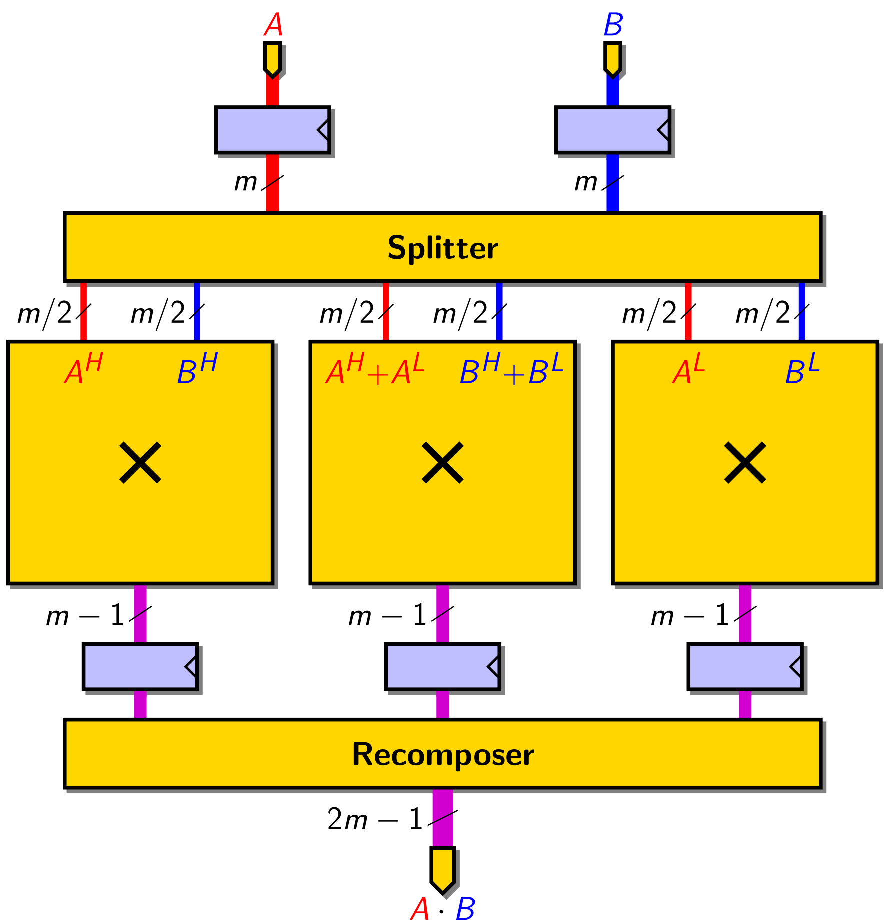
A parallel Karatsuba-Ofman multiplier



- fully parallel: all sub-products are computed in parallel

A parallel Karatsuba-Ofman multiplier



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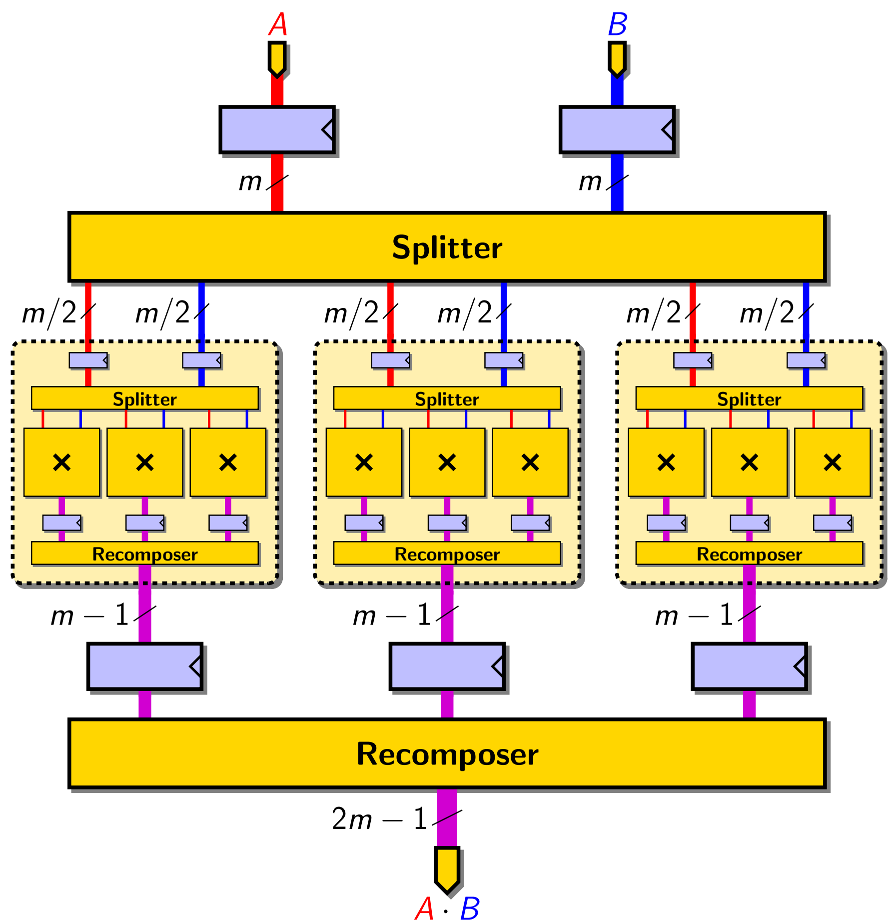
A parallel Karatsuba-Ofman multiplier



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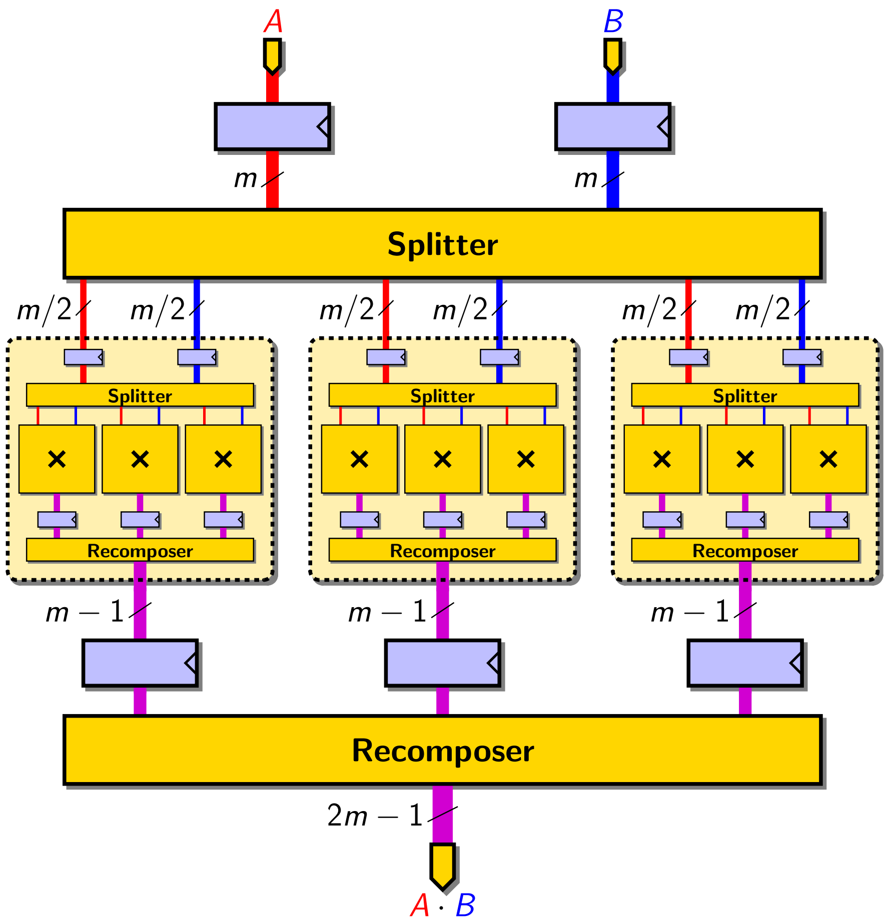

- fully parallel: all sub-products are computed in parallel
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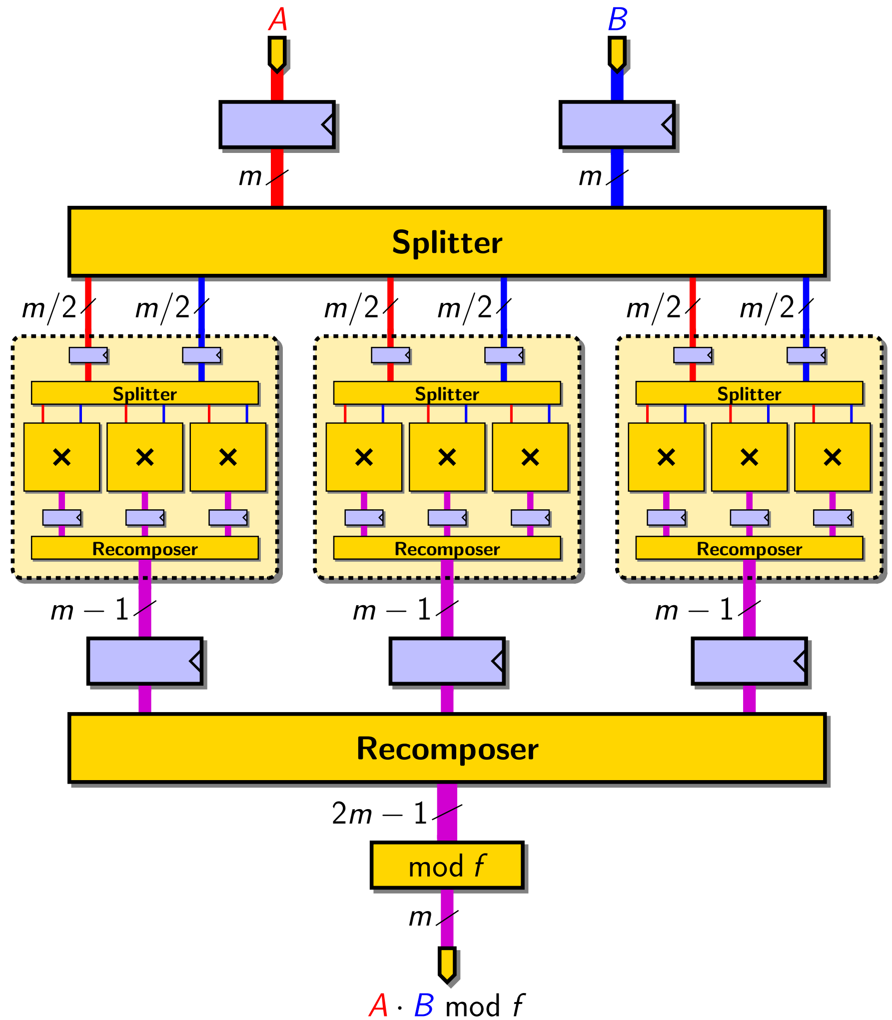
A parallel Karatsuba-Ofman multiplier



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- pipelined architecture: higher clock frequency, one product per cycle
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- support for other variants: odd-even split, 3-way split, ...
- final reduction modulo the irreducible polynomial f

Accelerating the η_T pairing

- Total cost: $17\times$, 10 Frobenius and $38+$ over $\mathbb{F}_{3^m}$ per iteration
- η_T coprocessor based on a single large multiplier:
  - parallel Karatsuba-Ofman architecture
  - 7-stage pipeline
  - one product per cycle

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  - 7-stage pipeline
  - one product per cycle
- Challenge: keep the multiplier busy at all times
- Careful scheduling to avoid pipeline bubbles (idle cycles):
  - ensure that multiplication operands are always available
  - avoid memory congestion issues

Data dependencies

① $x_Q \leftarrow x_Q^9 \pm 1$; $y_Q \leftarrow -y_Q^9$

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$
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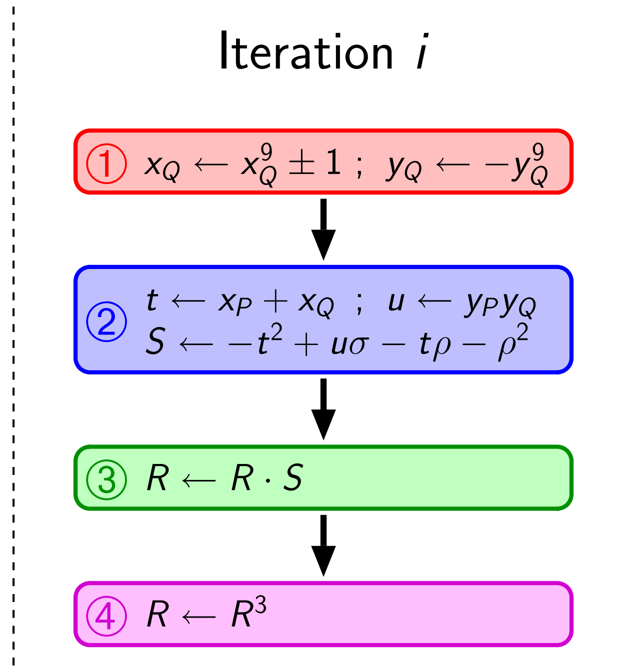
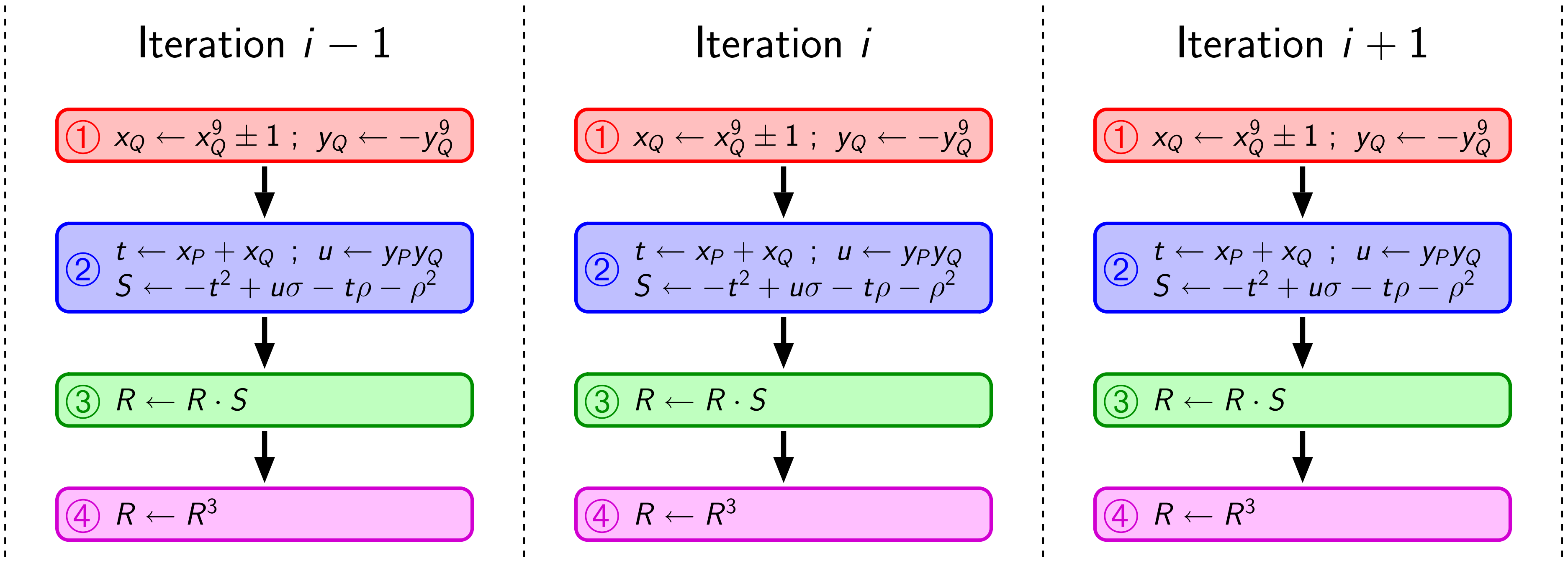
- Sequential dependencies between the tasks

Data dependencies



- Sequential dependencies between the tasks in each iteration

Data dependencies



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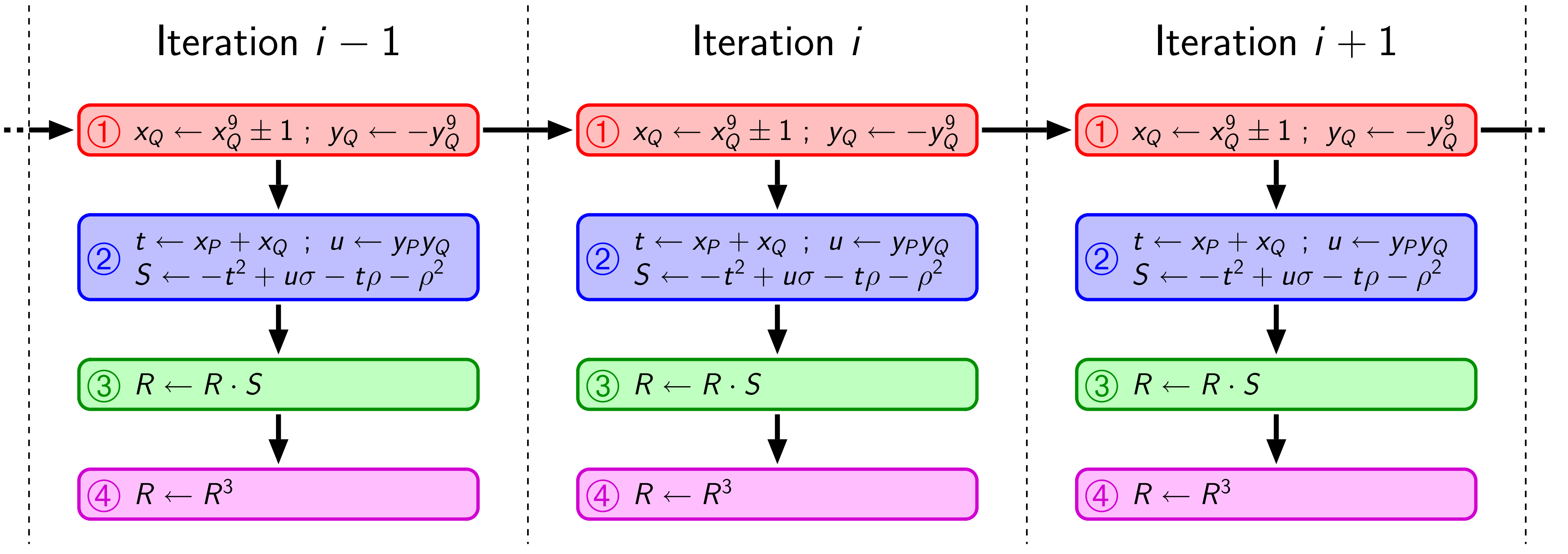

- Sequential dependencies between the tasks in each iteration
- Dependencies between consecutive iterations

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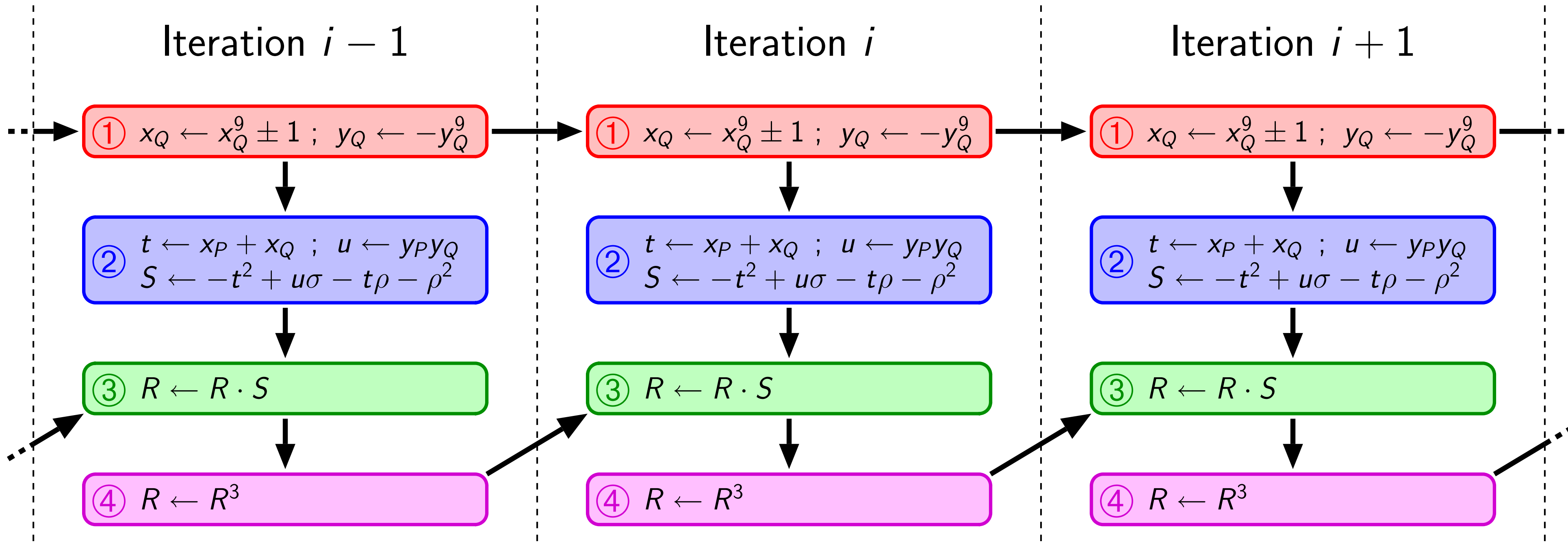

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Task scheduling

Task scheduling

① $x_Q \leftarrow x_Q^9 \pm 1$; $y_Q \leftarrow -y_Q^9$

Task scheduling

① 4 Frobenius, 2 +

Task scheduling

Task scheduling

①

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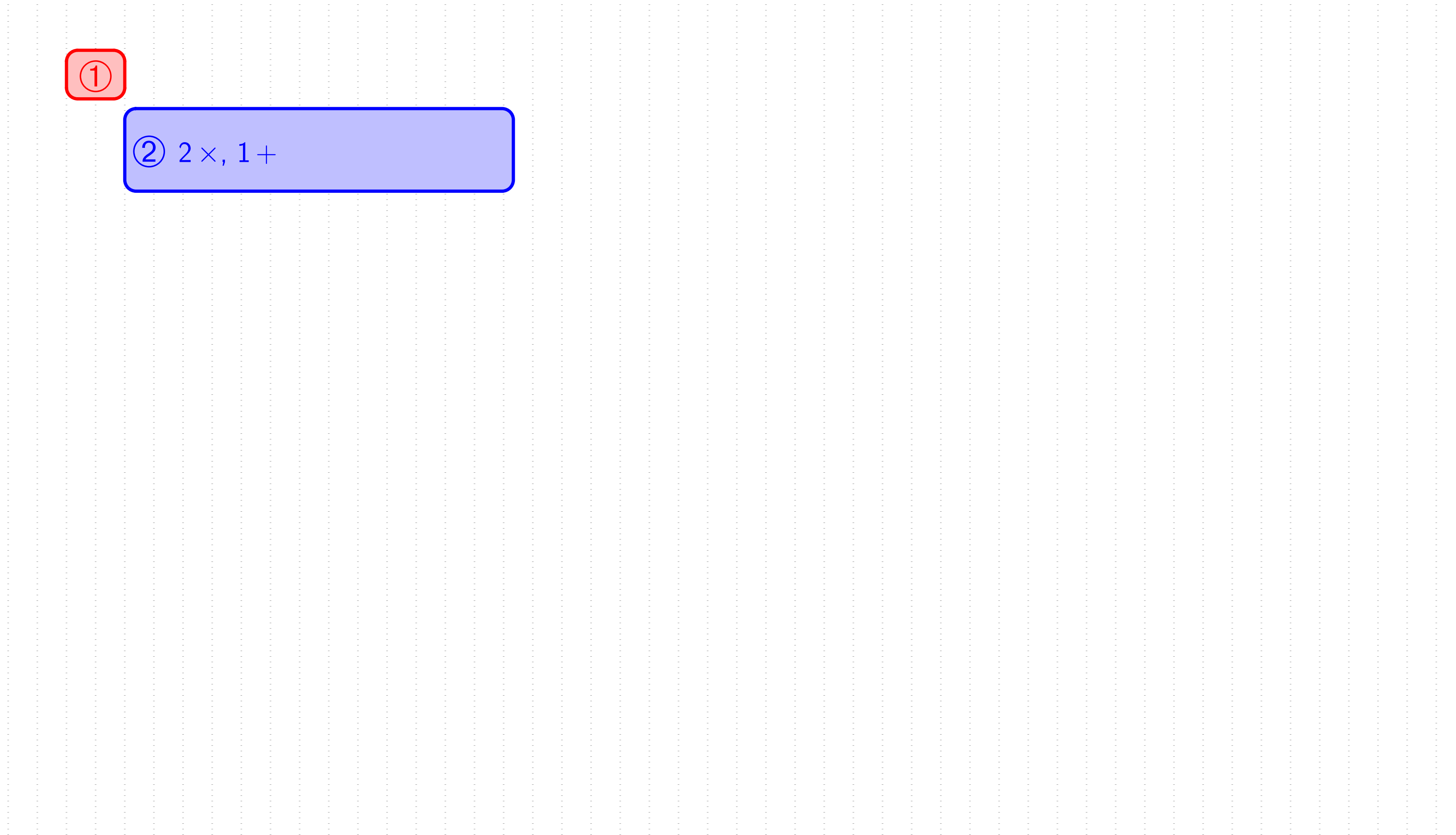
Task scheduling



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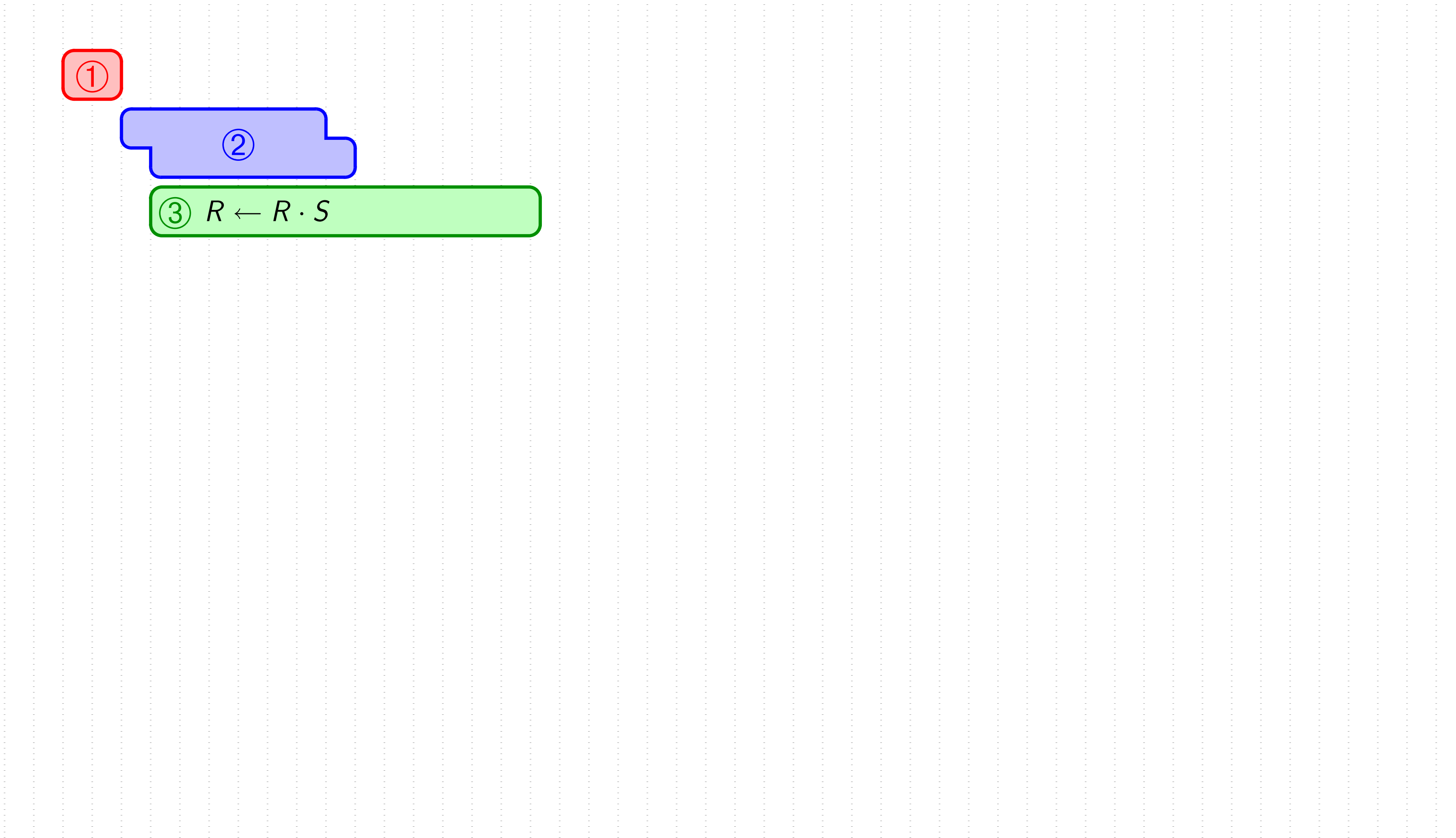

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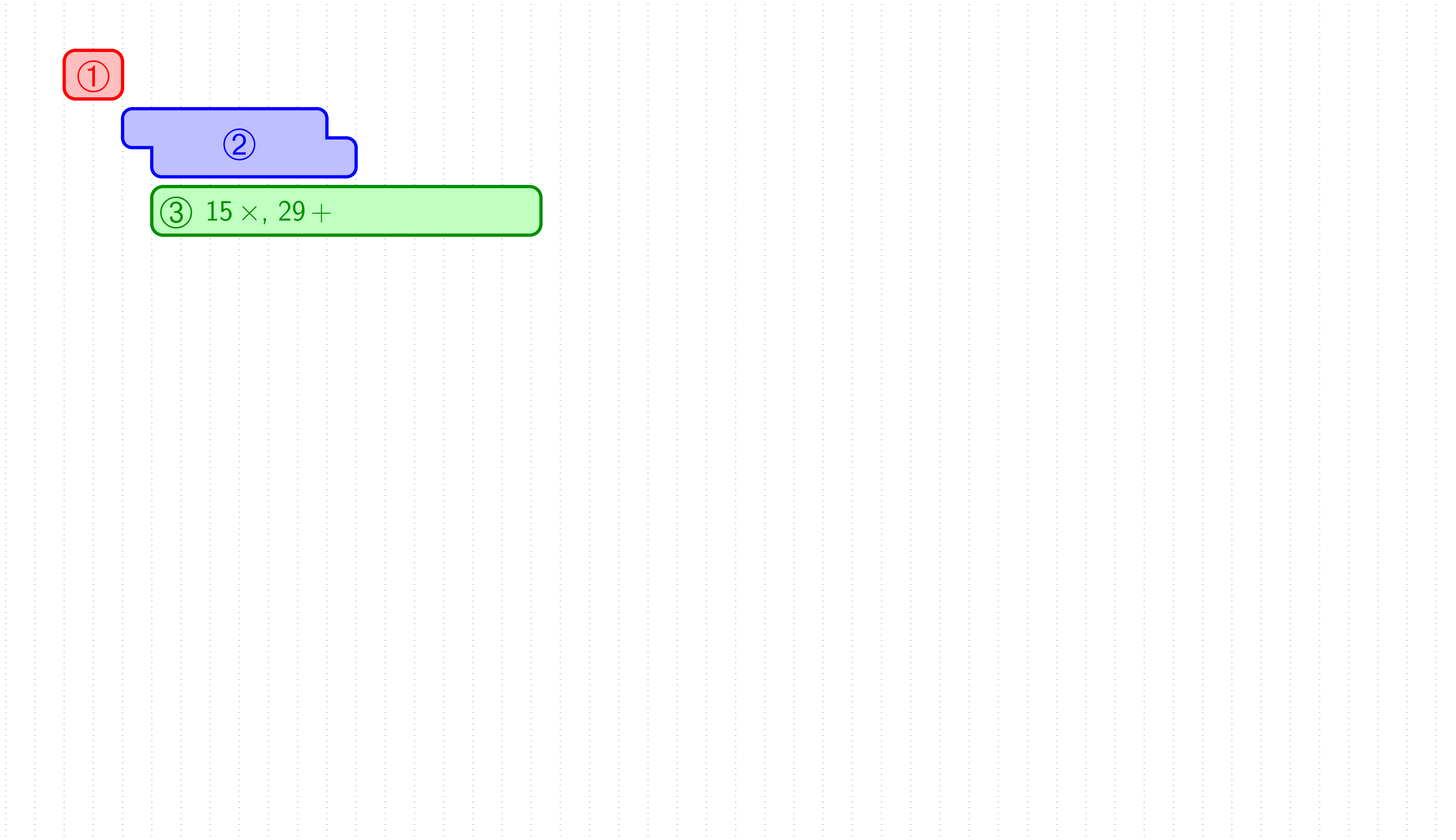

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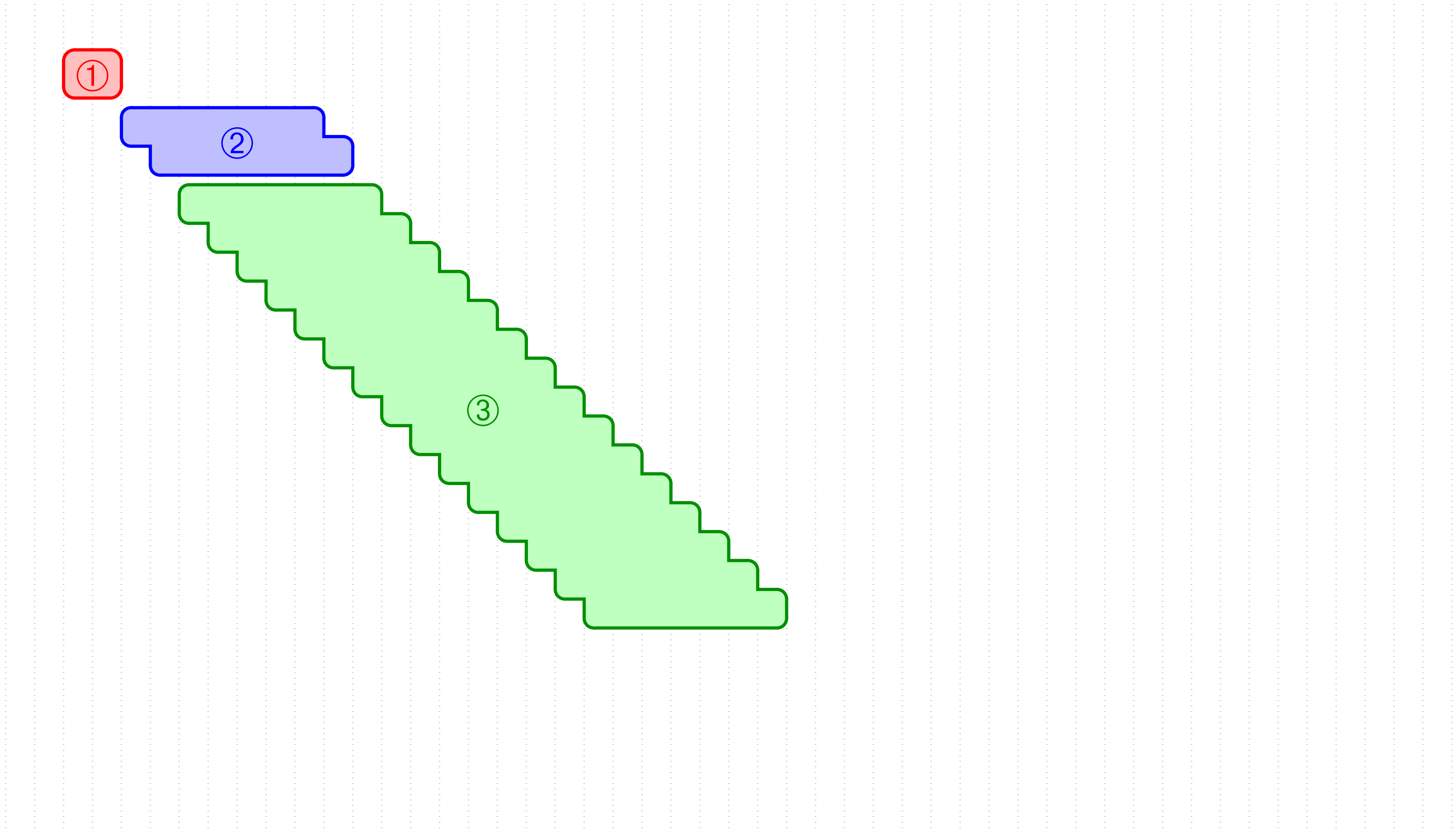

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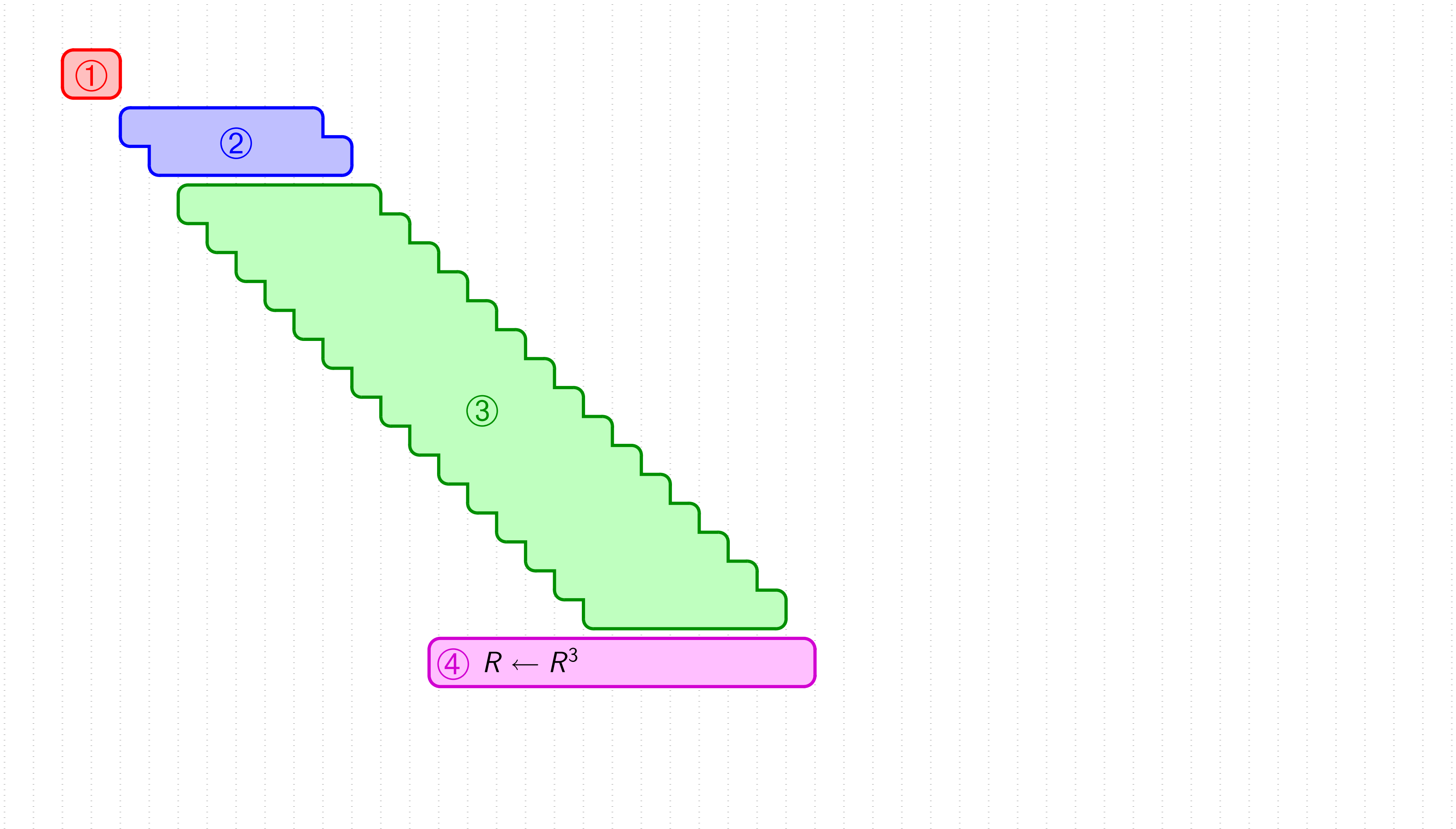

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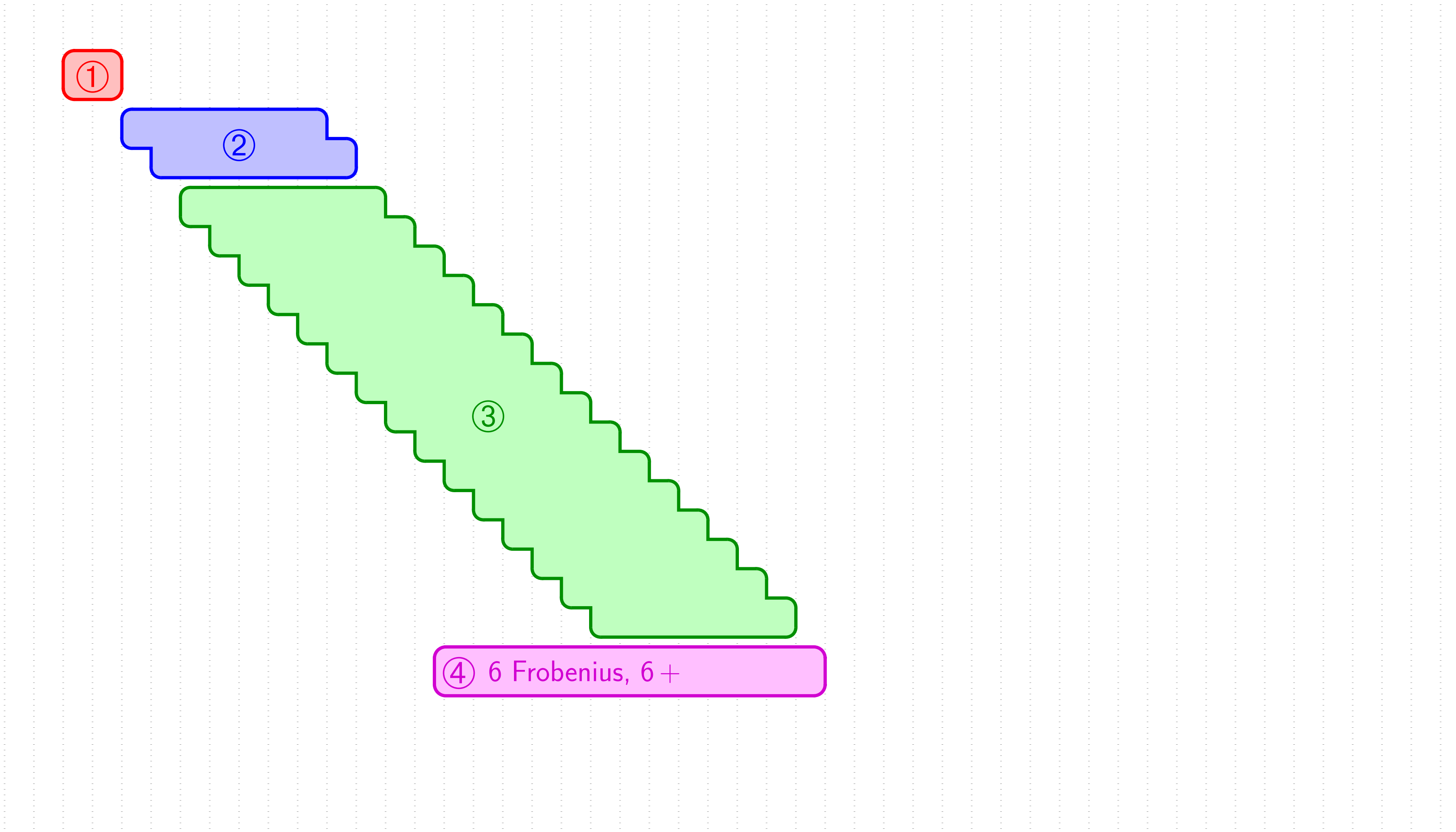

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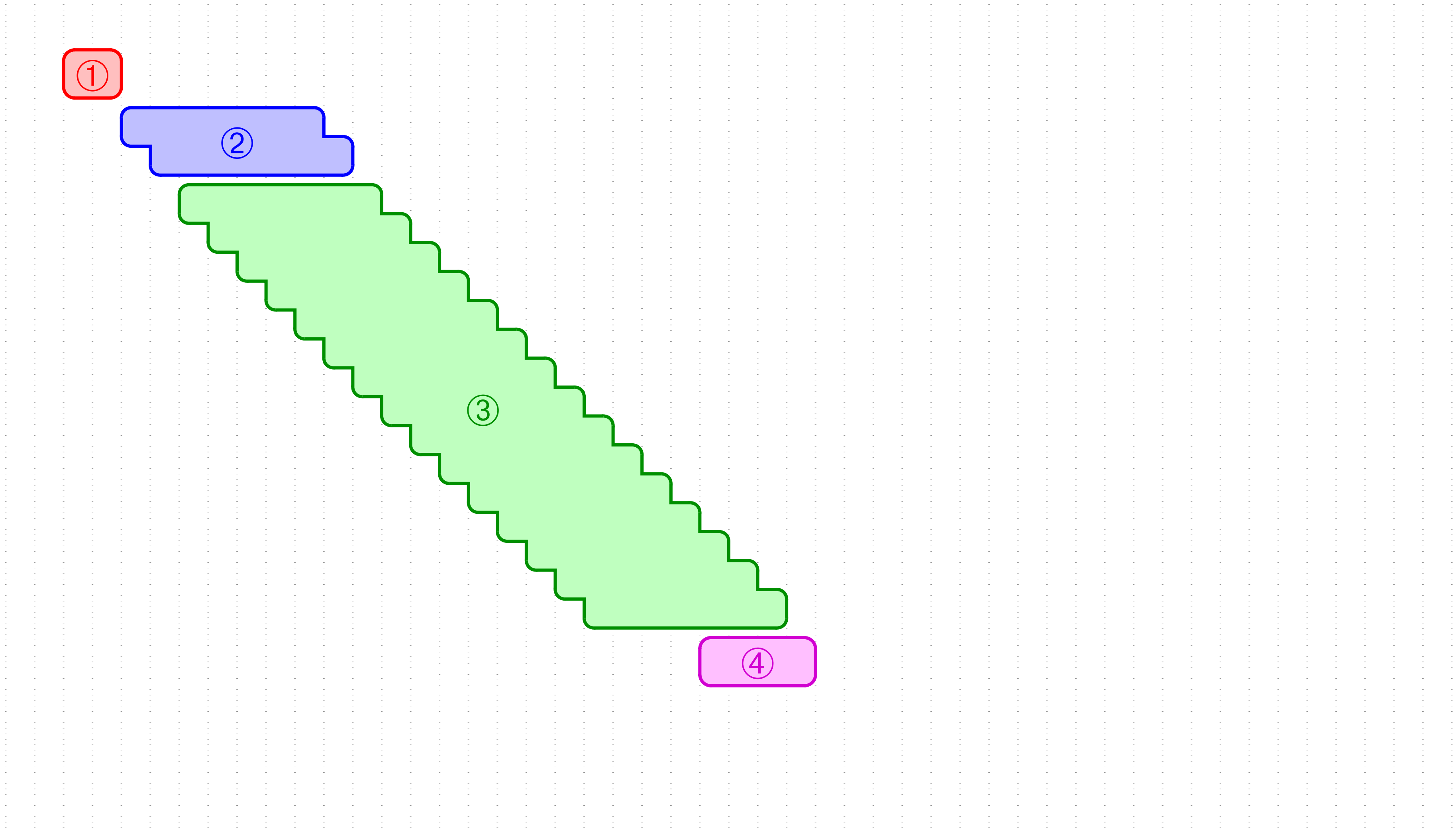

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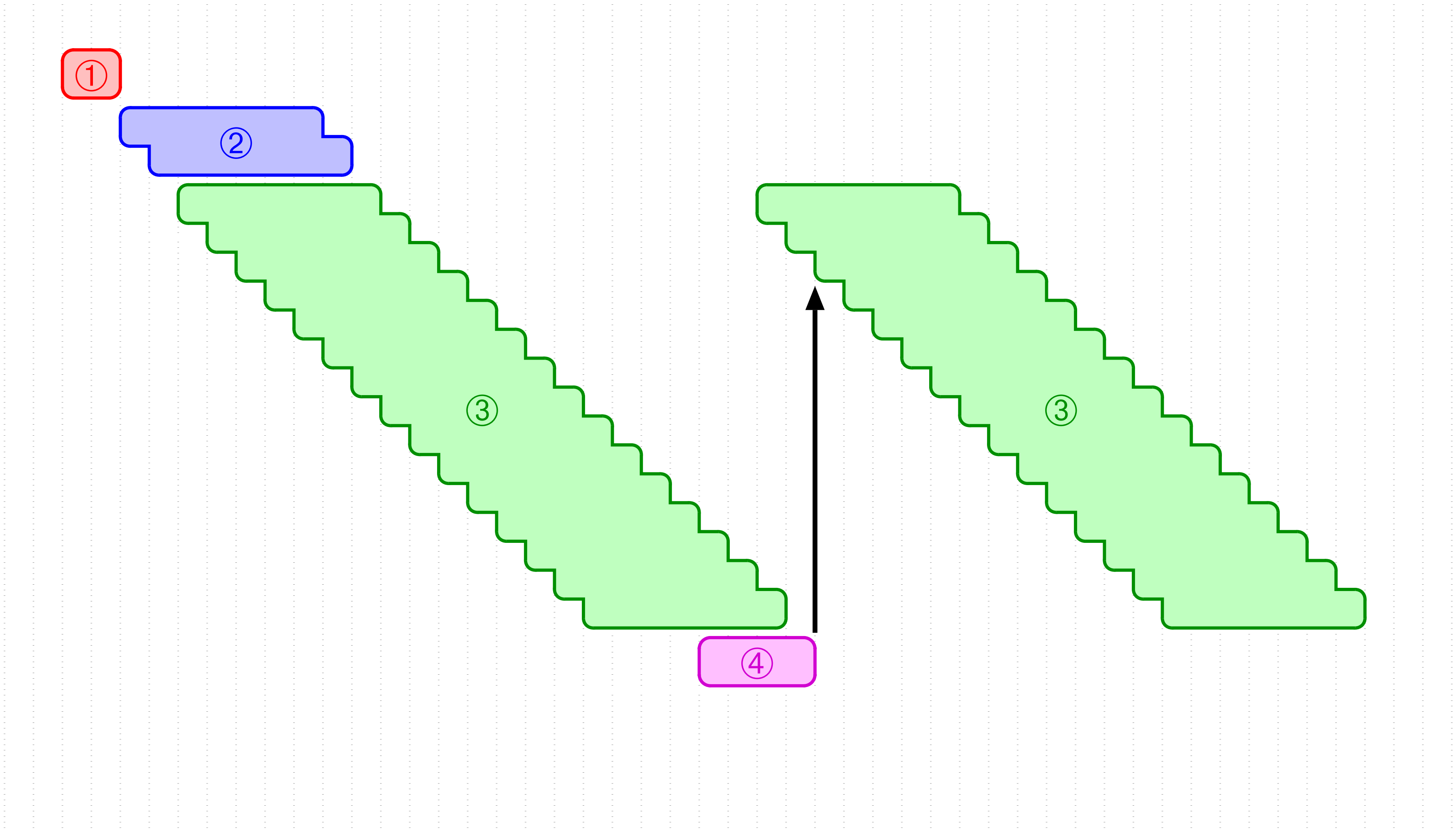

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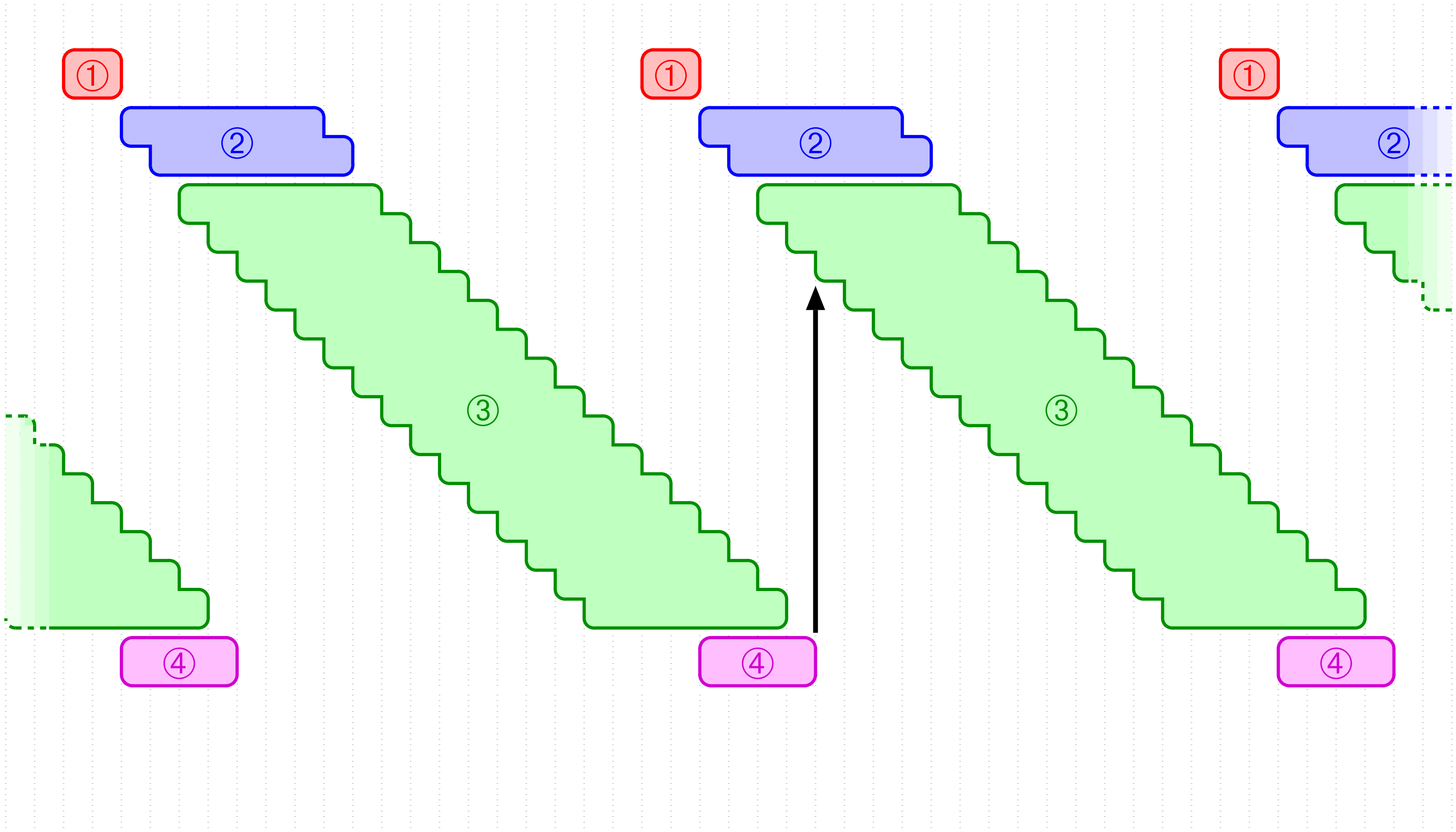

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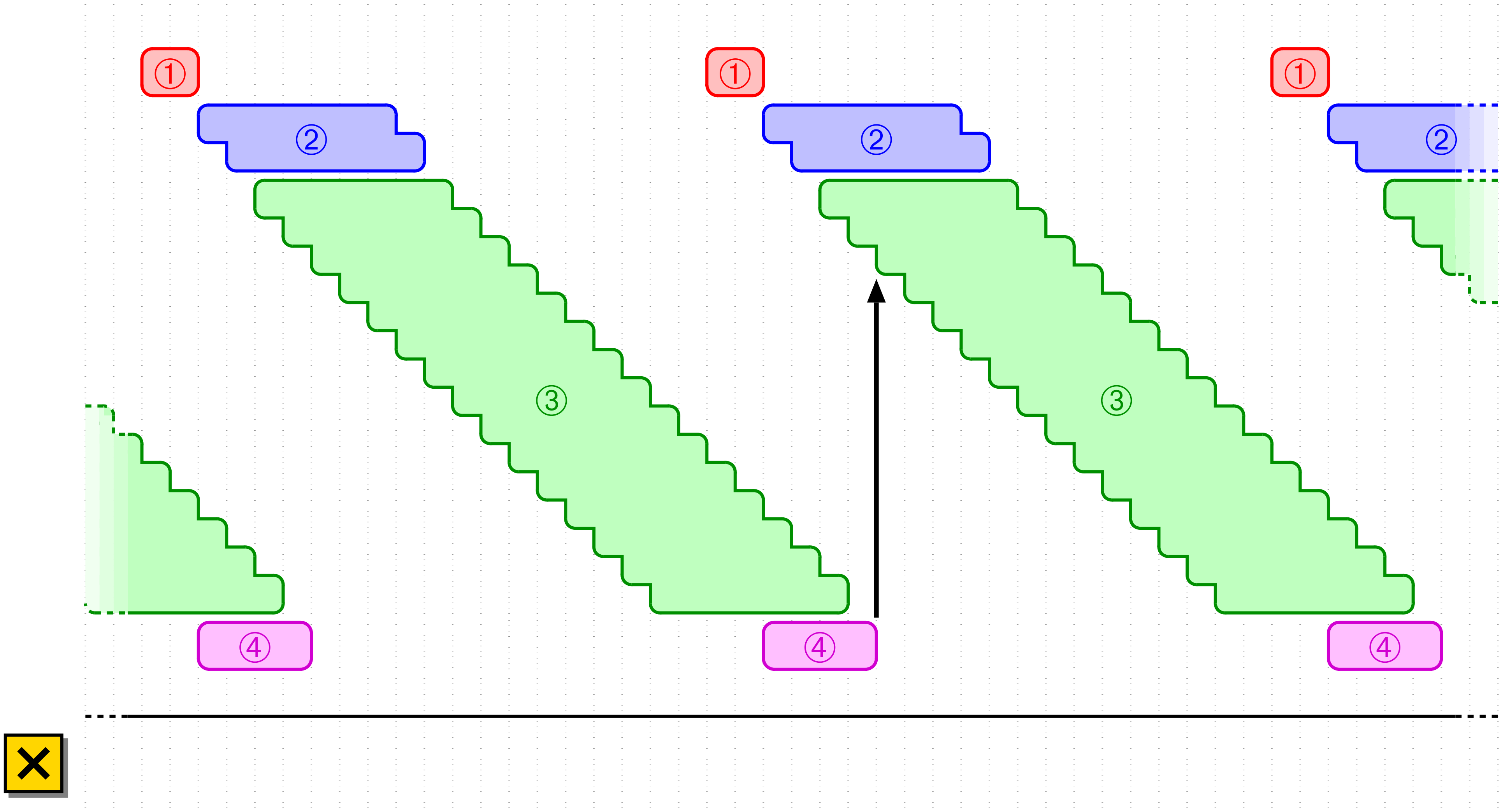

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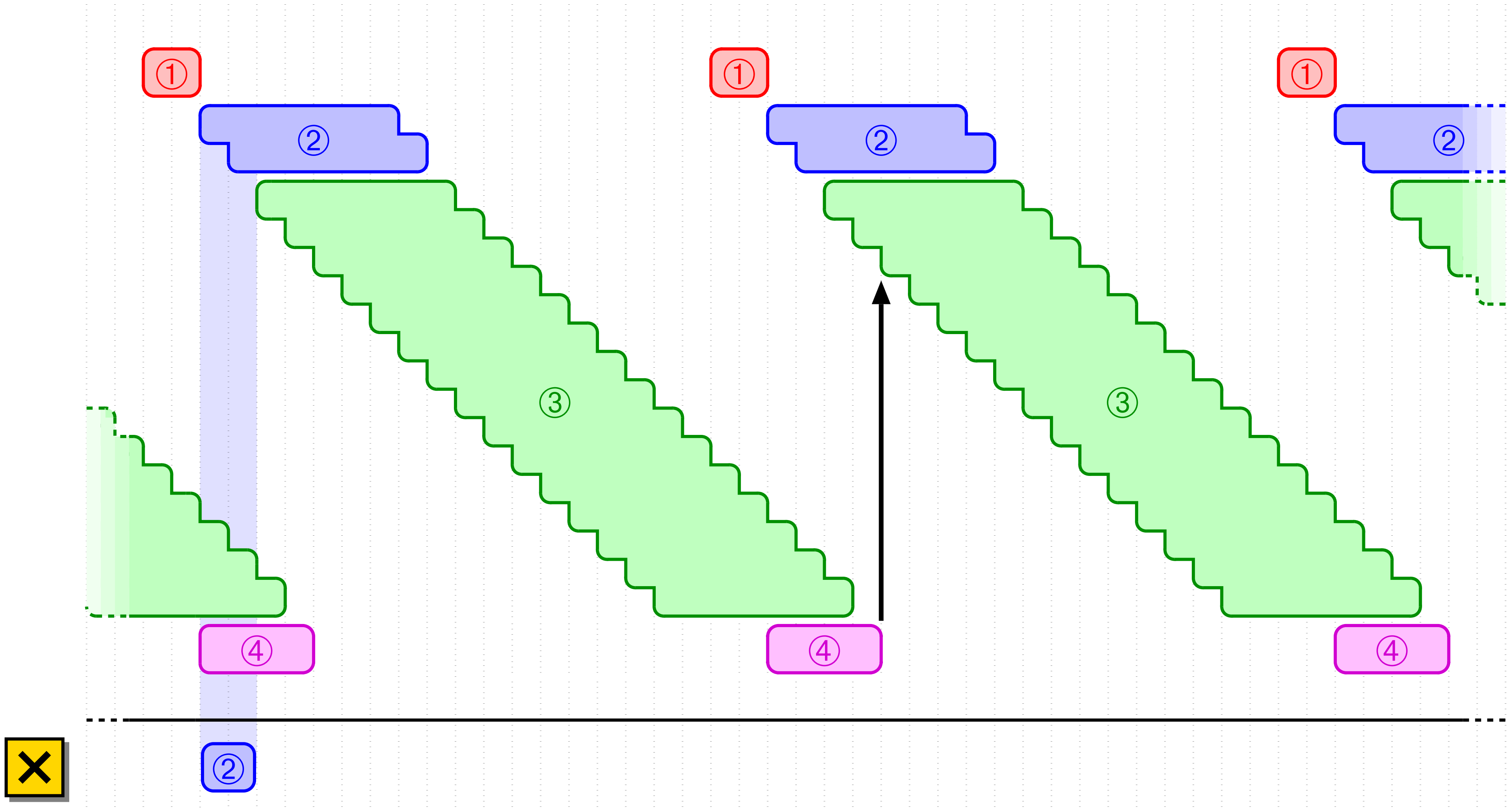

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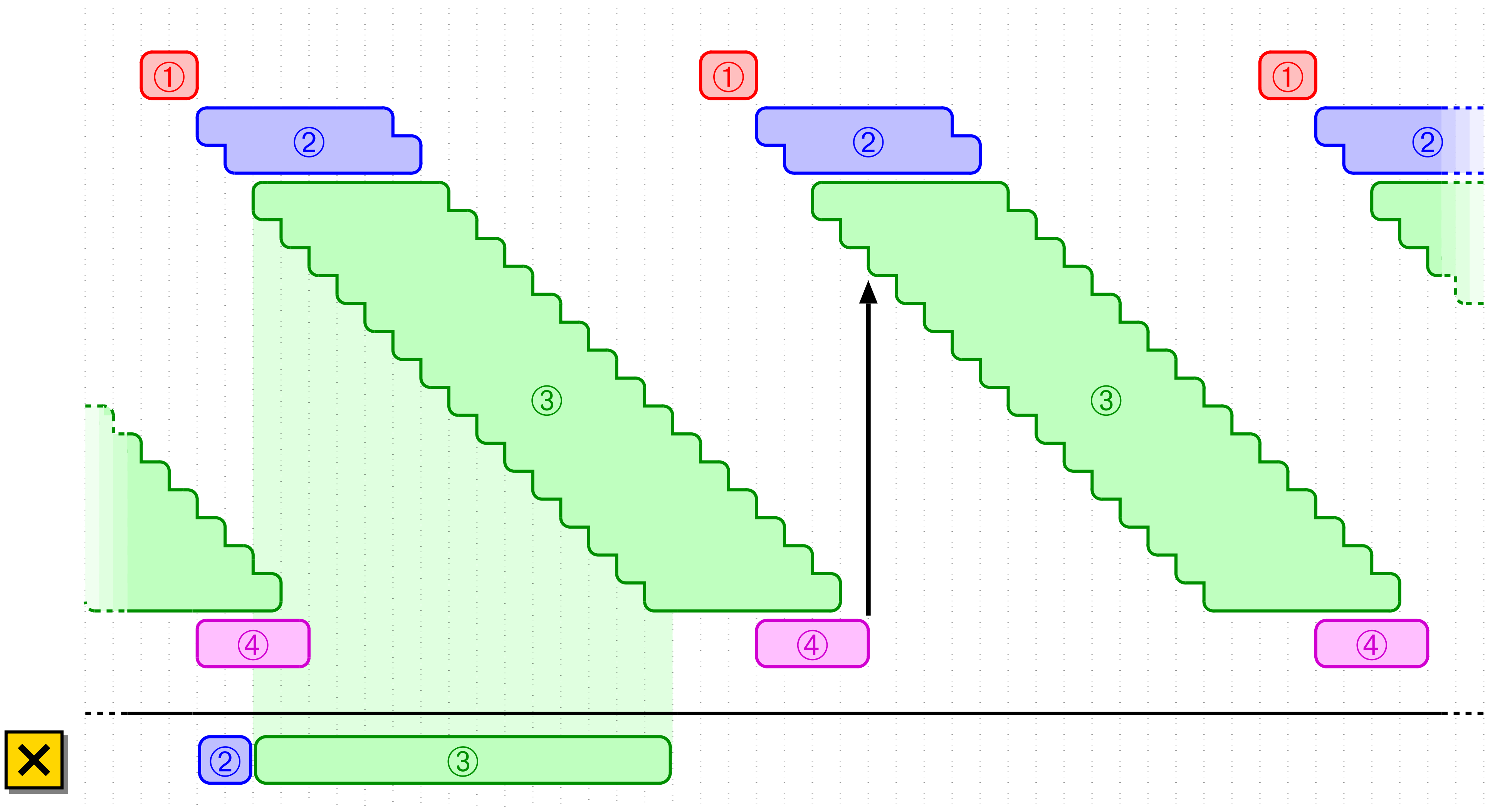

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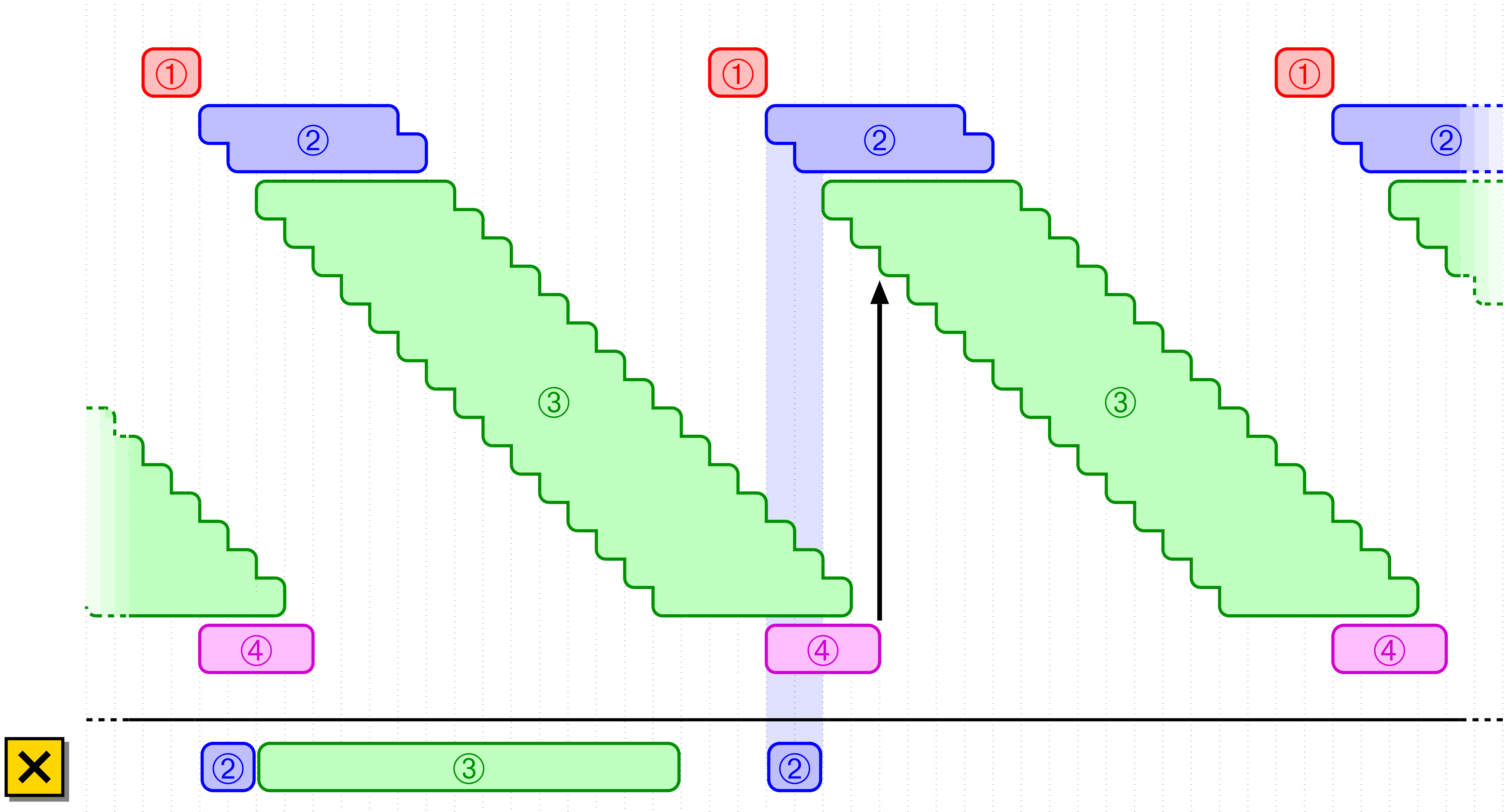

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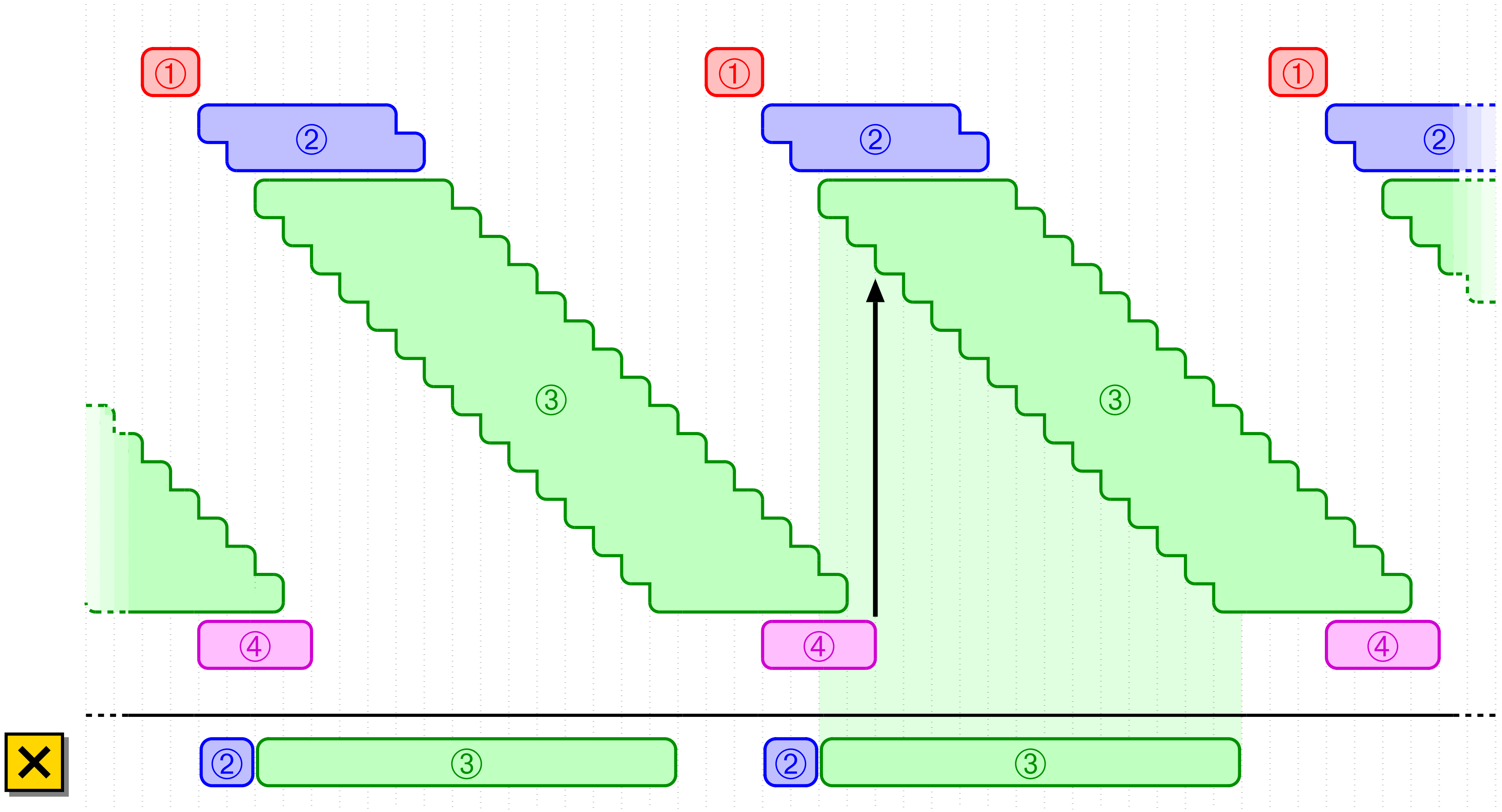

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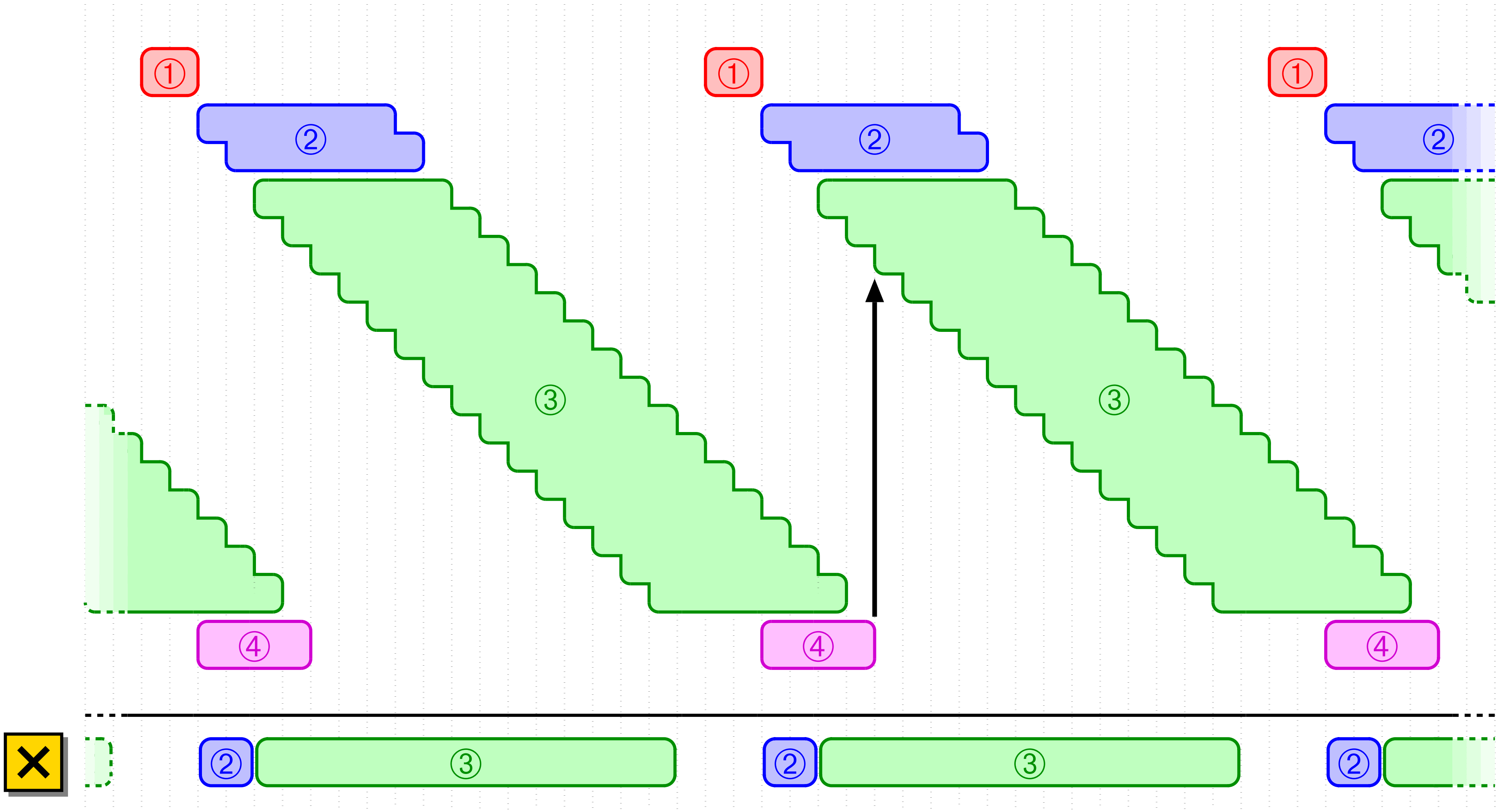
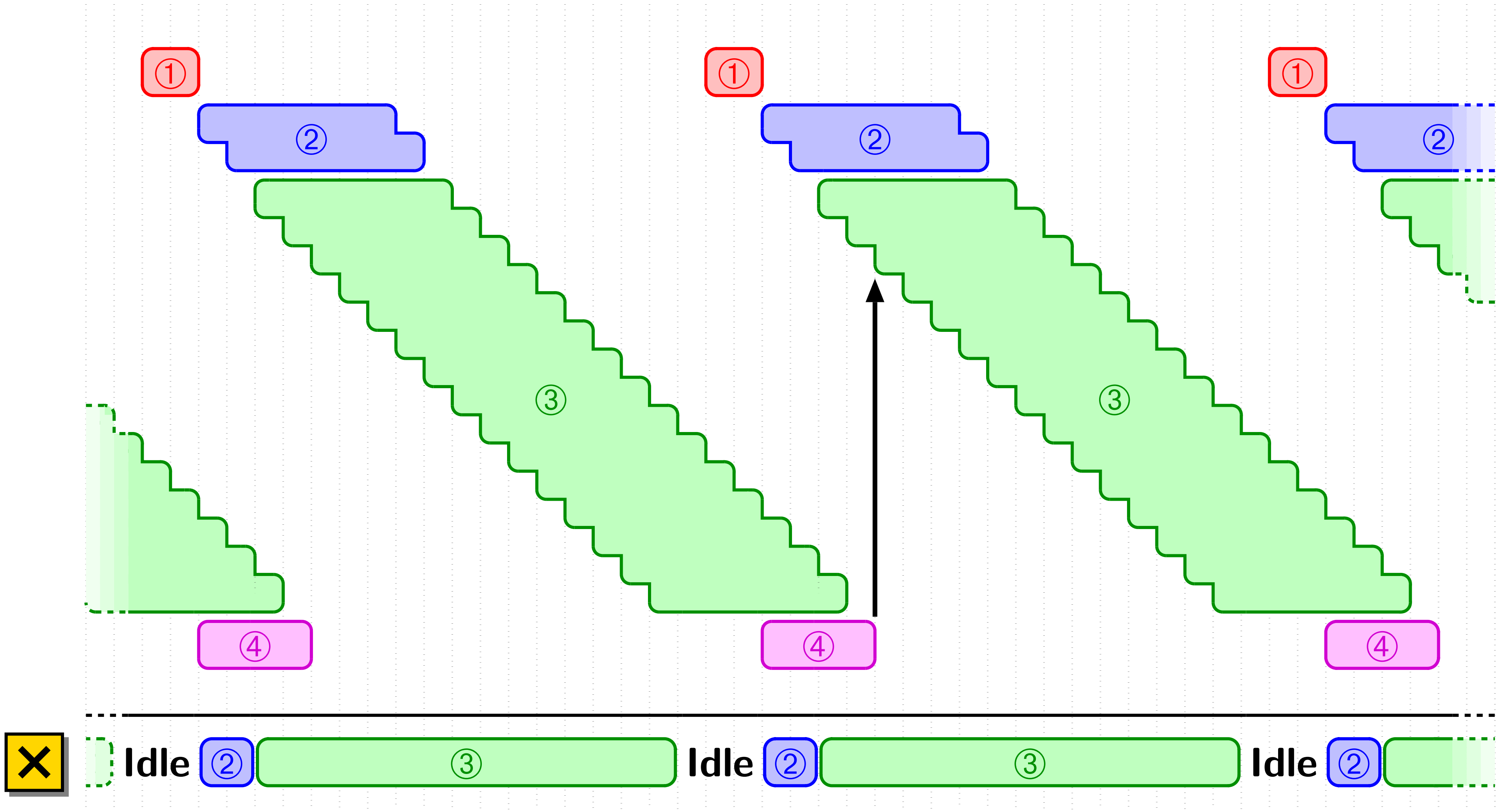
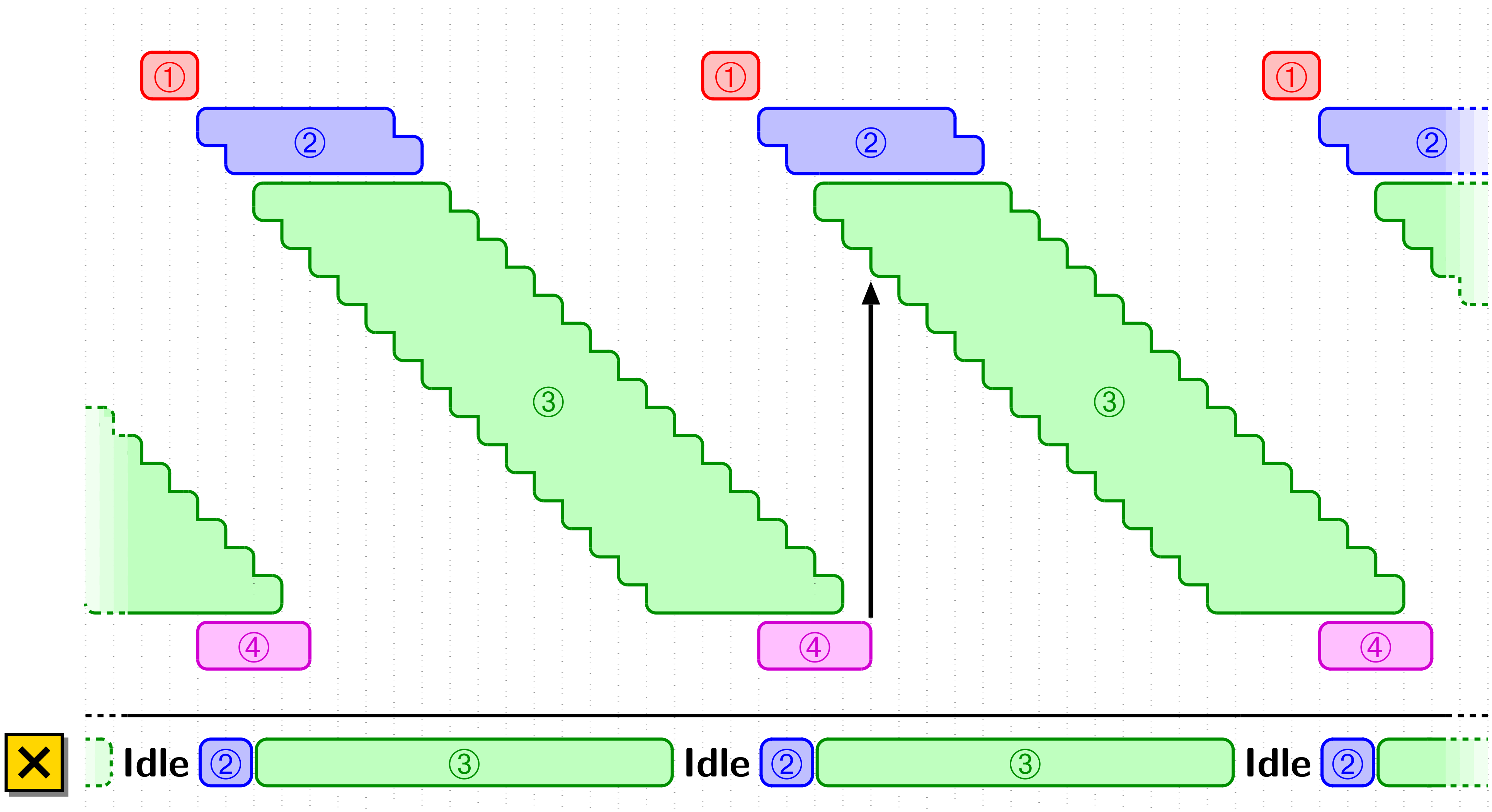


Task scheduling

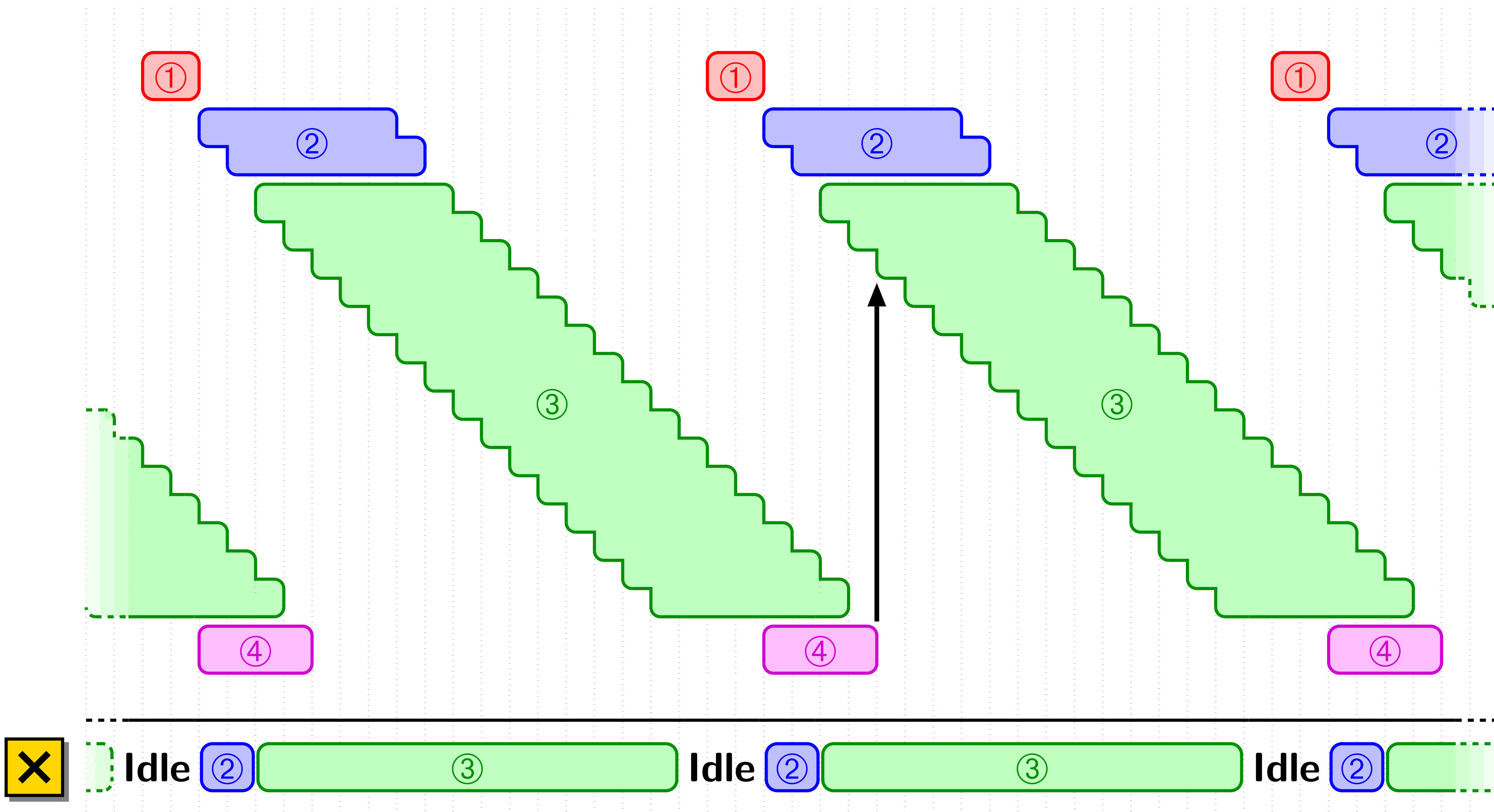


Task scheduling



- The task dependency ④ → ③ forces idle cycles into the multiplier pipeline

Task scheduling



- The task dependency ④ → ③ forces idle cycles into the multiplier pipeline
- The considered η_T pairing algorithm is not suited for parallel implementation

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \longrightarrow \mathbb{F}_{3^{6m}}^{\times}$$

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

① $x_Q \leftarrow x_Q^9 \pm 1 \; ; \; y_Q \leftarrow -y_Q^9$

② $t \leftarrow x_P + x_Q \; ; \; u \leftarrow y_P y_Q$
$S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

④ $R \leftarrow R^3$

end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \to \mathbb{F}_{3^{6m}}^{\times}$$

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

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A modified algorithm

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$x_Q \leftarrow x_Q^{9} \pm 1$; $y_Q \leftarrow -y_Q^{9}$

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$
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end for

A modified algorithm

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for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

① $x_P \leftarrow x_P$; $y_P \leftarrow y_P$
$x_Q \leftarrow x_Q^{\mathbf{9}} \pm 1$; $y_Q \leftarrow -y_Q^{\mathbf{9}}$

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$
$S \leftarrow -t^2 + u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

④ $R \leftarrow R^{\mathbf{3}}$

end for

⇒

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

① $x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$
$x_Q \leftarrow x_Q^{\mathbf{3}}$; $y_Q \leftarrow y_Q^{\mathbf{3}}$

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$
$S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

④ $R \leftarrow R$

end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \longrightarrow \mathbb{F}_{3^{6m}}^\times$$

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

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$x_Q \leftarrow x_Q^{9} \pm 1$; $y_Q \leftarrow -y_Q^{9}$

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③ $R \leftarrow R \cdot S$

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end for

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

① $x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$
$x_Q \leftarrow x_Q^{3}$; $y_Q \leftarrow y_Q^{3}$

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A modified algorithm

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end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \to \mathbb{F}_{3^{6m}}^{\times}$$

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① $x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$
$x_Q \leftarrow x_Q^3$; $y_Q \leftarrow y_Q^3$

2 inv. Frobenius
2 Frobenius
$(\mathbb{F}_{3^m})$

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$
$S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

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end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^{\times}$$

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

① $x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$ — 2 inv. Frobenius $(\mathbb{F}_{3^m})$
$x_Q \leftarrow x_Q^3$; $y_Q \leftarrow y_Q^3$ — 2 Frobenius

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$ — $2\times$, $1+$ $(\mathbb{F}_{3^m})$
$S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

③ $R \leftarrow R \cdot S$

end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \longrightarrow \mathbb{F}_{3^{6m}}^{\times}$$

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②	$t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$ $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$	$2\times$, $1+$	$(\mathbb{F}_{3^m})$
③	$R \leftarrow R \cdot S$	$1\times$	$(\mathbb{F}_{3^{6m}})$

end for

A modified algorithm

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②	$t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$ $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$	$2\times$, $1+$	$(\mathbb{F}_{3^m})$
③	$R \leftarrow R \cdot S$	$15\times$, $29+$	$(\mathbb{F}_{3^m})$

end for

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \to \mathbb{F}_{3^{6m}}^{\times}$$

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

①	$x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$ $x_Q \leftarrow x_Q^3$; $y_Q \leftarrow y_Q^3$	2 inv. Frobenius 2 Frobenius	$(\mathbb{F}_{3^m})$
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③	$R \leftarrow R \cdot S$	$15\times$, $29+$	$(\mathbb{F}_{3^m})$

end for

- Modified algorithm: $17\times$, 2 Frobenius, 2 inverse Frobenius and $30+$ over $\mathbb{F}_{3^m}$

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^{\times}$$

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- Modified algorithm: $17\times$, 2 Frobenius, 2 inverse Frobenius and $30+$ over $\mathbb{F}_{3^m}$
- Previous algorithm: $17\times$, 10 Frobenius and $38+$

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^{\times}$$

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

①	$x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$ $x_Q \leftarrow x_Q^3$; $y_Q \leftarrow y_Q^3$	2 inv. Frobenius 2 Frobenius	$(\mathbb{F}_{3^m})$
②	$t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$ $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$	$2\times$, $1+$	$(\mathbb{F}_{3^m})$
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end for

- Modified algorithm: $17\times$, 2 Frobenius, 2 inverse Frobenius and $30+$ over $\mathbb{F}_{3^m}$
- Previous algorithm: $17\times$, 10 Frobenius and $38+$
- Cost of the inverse Frobenius?

A modified algorithm

$$\eta_T : E(\mathbb{F}_{3^m})[\ell] \times E(\mathbb{F}_{3^m})[\ell] \rightarrow \mathbb{F}_{3^{6m}}^{\times}$$

for $i \leftarrow 0$ **to** $(m-1)/2$ **do**

①	$x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$ $x_Q \leftarrow x_Q^3$; $y_Q \leftarrow y_Q^3$	2 inv. Frobenius 2 Frobenius	$(\mathbb{F}_{3^m})$
②	$t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$ $S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$	$2\times$, $1+$	$(\mathbb{F}_{3^m})$
③	$R \leftarrow R \cdot S$	$15\times$, $29+$	$(\mathbb{F}_{3^m})$

end for

- Modified algorithm: $17\times$, 2 Frobenius, 2 inverse Frobenius and $30+$ over $\mathbb{F}_{3^m}$
- Previous algorithm: $17\times$, 10 Frobenius and $38+$
- Cost of the inverse Frobenius? Same as the Frobenius

Data dependencies

Iteration $i-1$

① $x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$
$x_Q \leftarrow x_Q^3$; $y_Q \leftarrow y_Q^3$

↓

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$
$S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

↓

③ $R \leftarrow R \cdot S$

Iteration i

① $x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$
$x_Q \leftarrow x_Q^3$; $y_Q \leftarrow y_Q^3$

↓

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$
$S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

↓

③ $R \leftarrow R \cdot S$

Iteration $i+1$

① $x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$
$x_Q \leftarrow x_Q^3$; $y_Q \leftarrow y_Q^3$

↓

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$
$S \leftarrow -t^2 \pm u\sigma - t\rho - \rho^2$

↓

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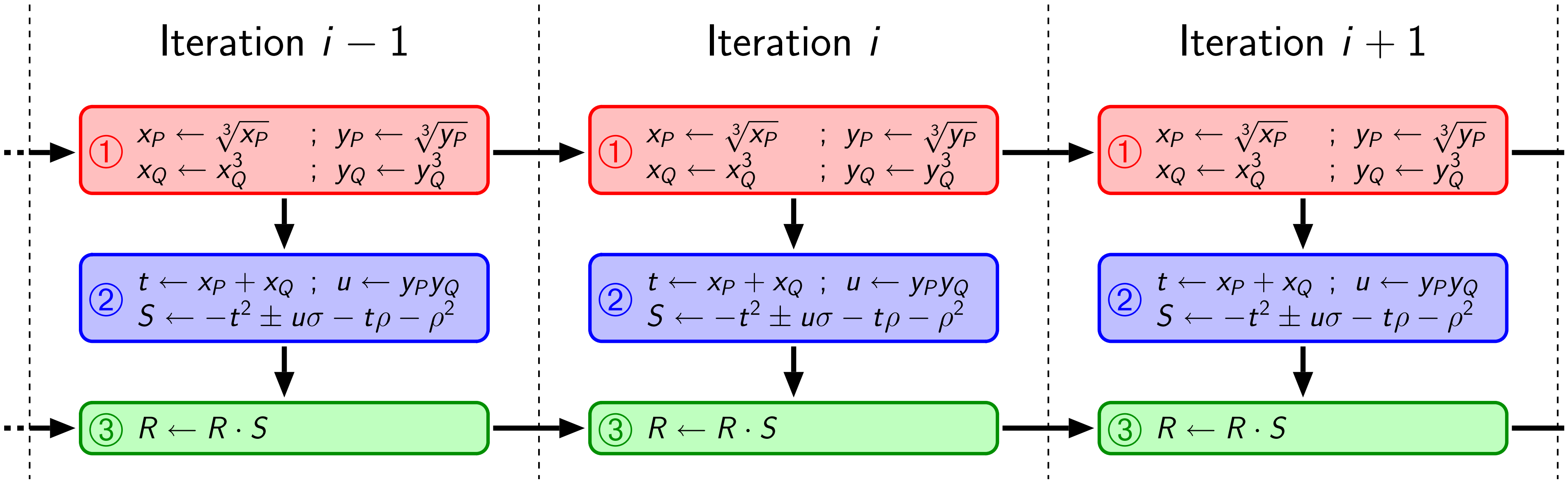
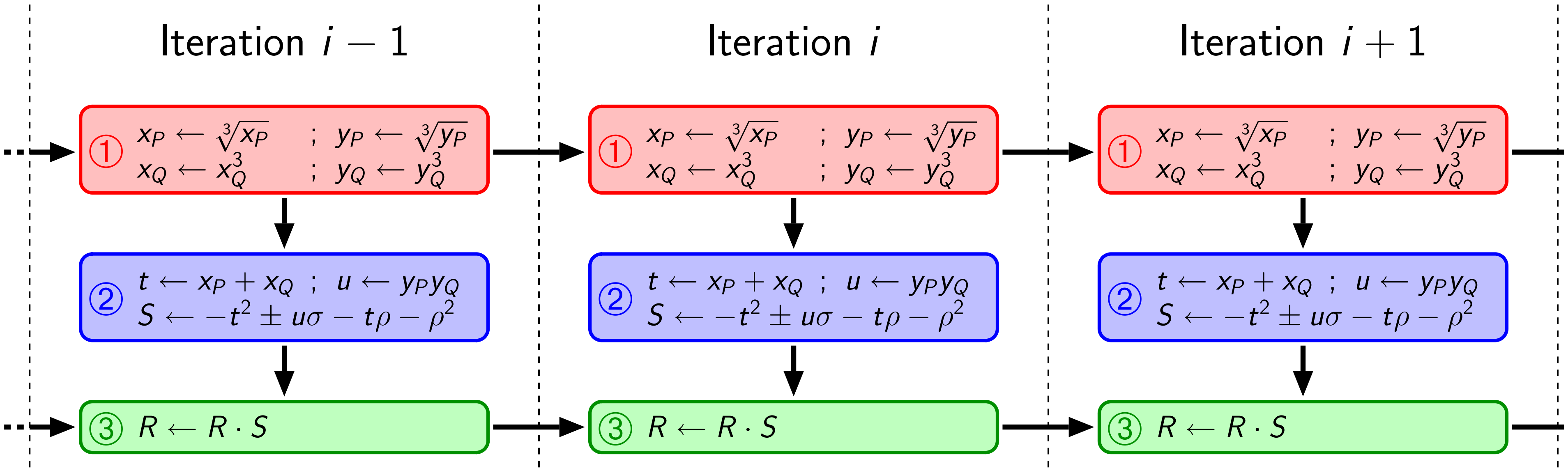
Data dependencies



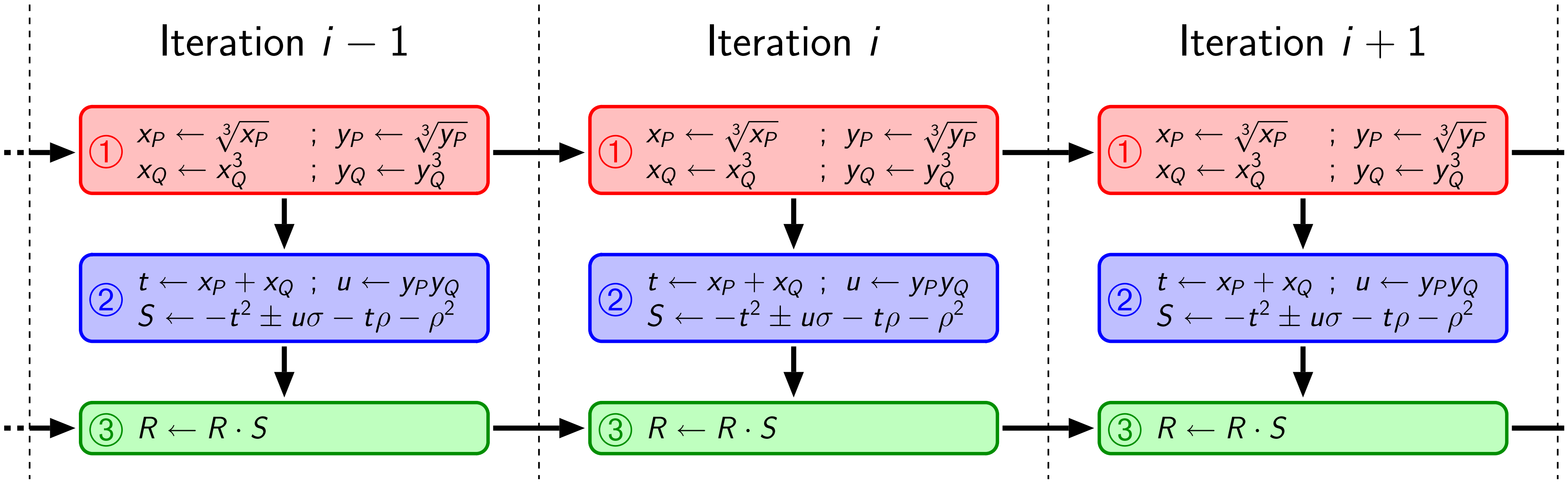
- Direct dependency ③ $\rightarrow$ ③ between consecutive iterations

Data dependencies



- Direct dependency ③ → ③ between consecutive iterations
  - avoid the scheduling bottleneck of task ④
  - better overlapping of successive tasks ③

Data dependencies



- Direct dependency ③ $\rightarrow$ ③ between consecutive iterations
  - avoid the scheduling bottleneck of task ④
  - better overlapping of successive tasks ③
  - hopefully tighter scheduling

Task scheduling

Task scheduling

① $x_P \leftarrow \sqrt[3]{x_P}$; $y_P \leftarrow \sqrt[3]{y_P}$
$x_Q \leftarrow x_Q^3$; $y_Q \leftarrow y_Q^3$

Task scheduling

Task scheduling

Task scheduling

①

② $t \leftarrow x_P + x_Q$; $u \leftarrow y_P y_Q$
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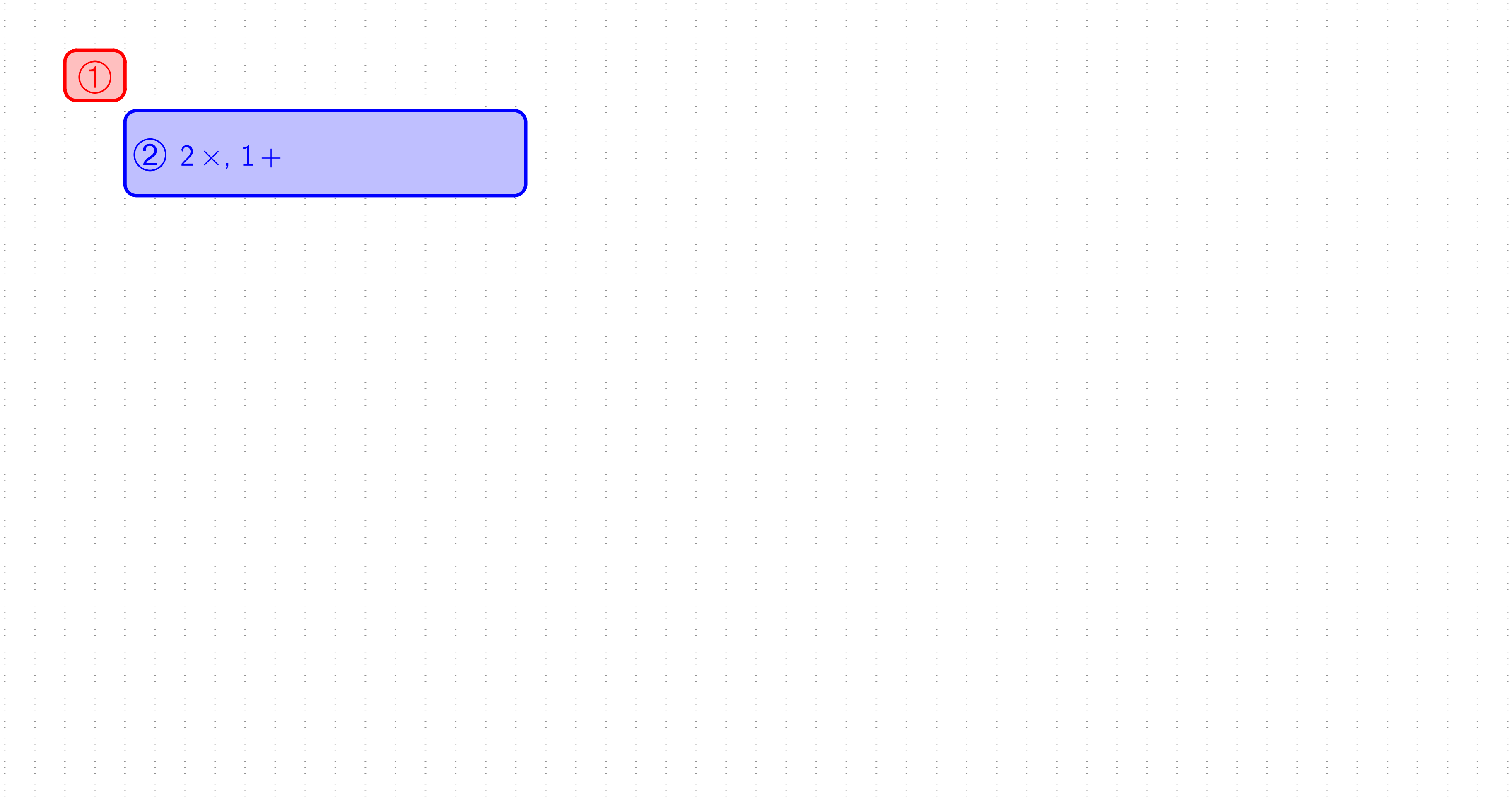
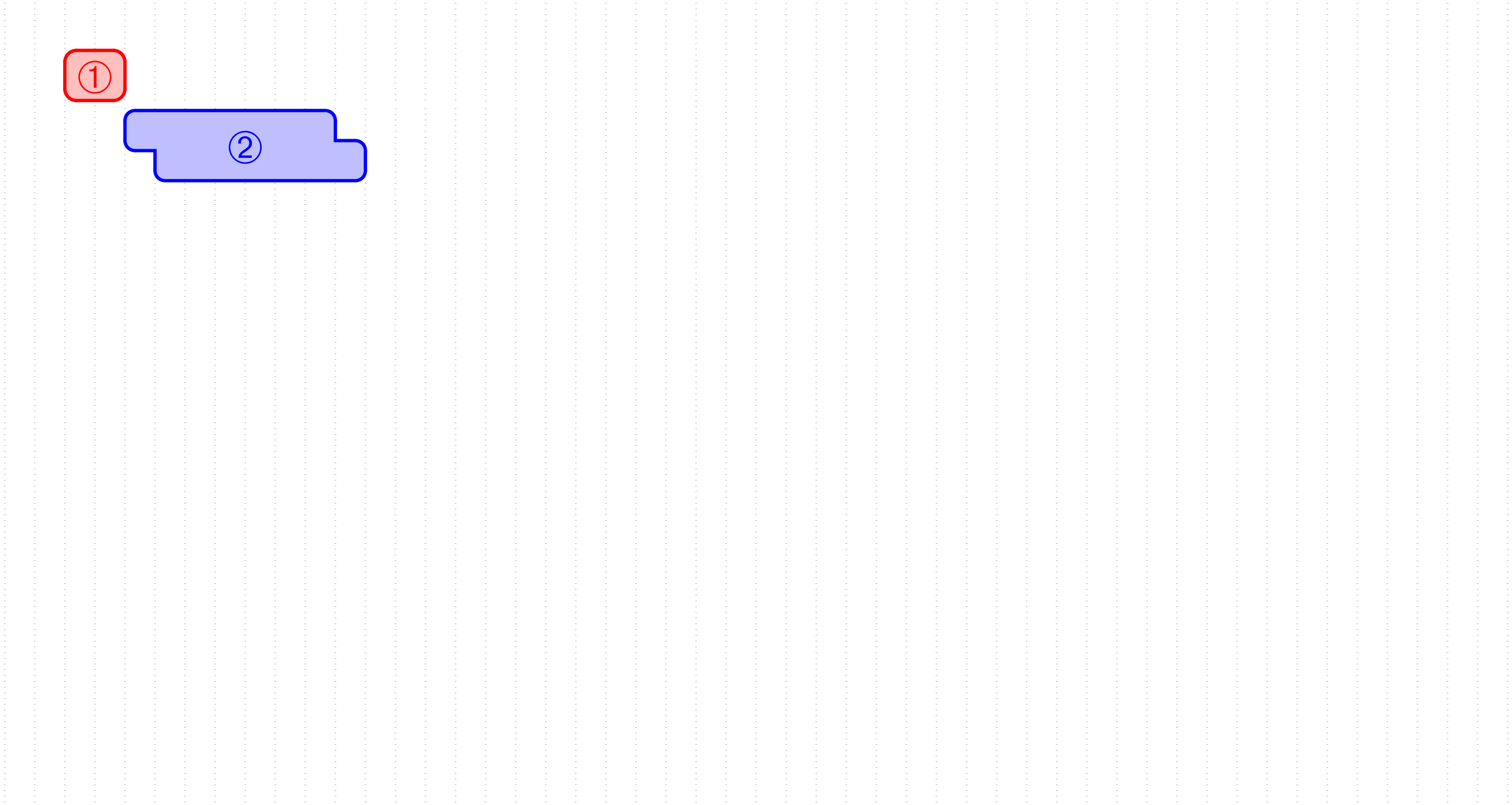
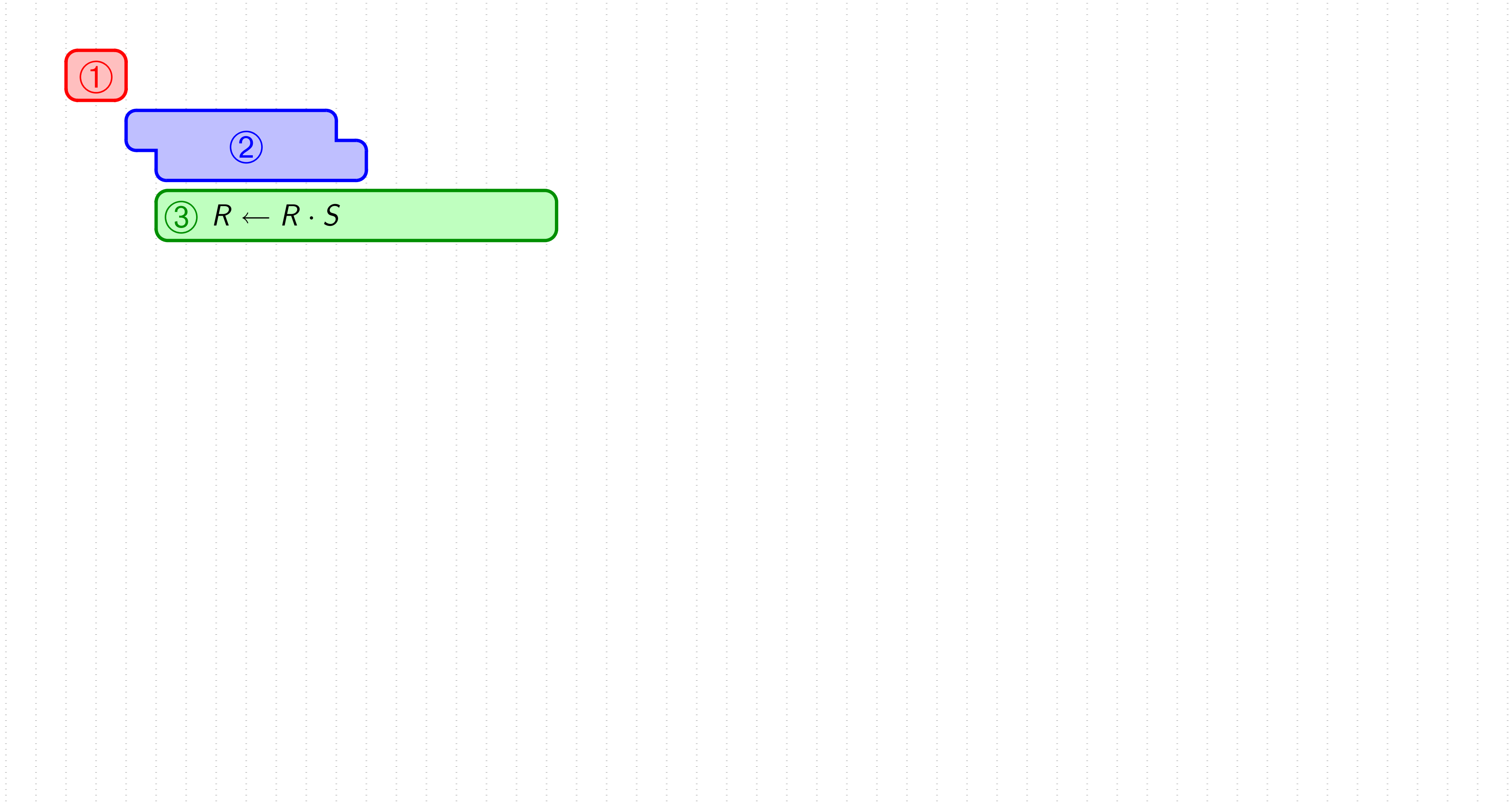
Task scheduling



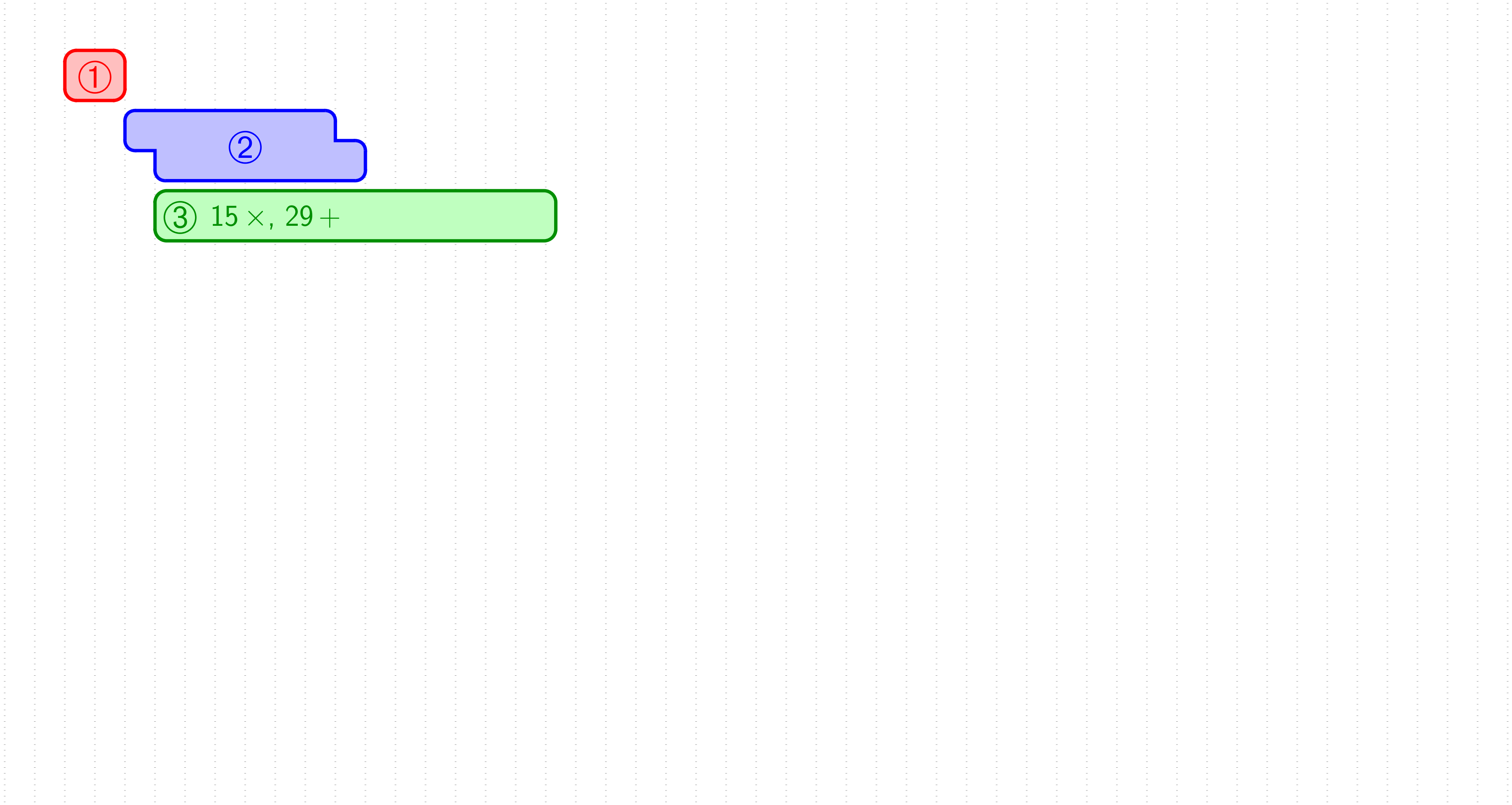
Task scheduling



Task scheduling



Task scheduling



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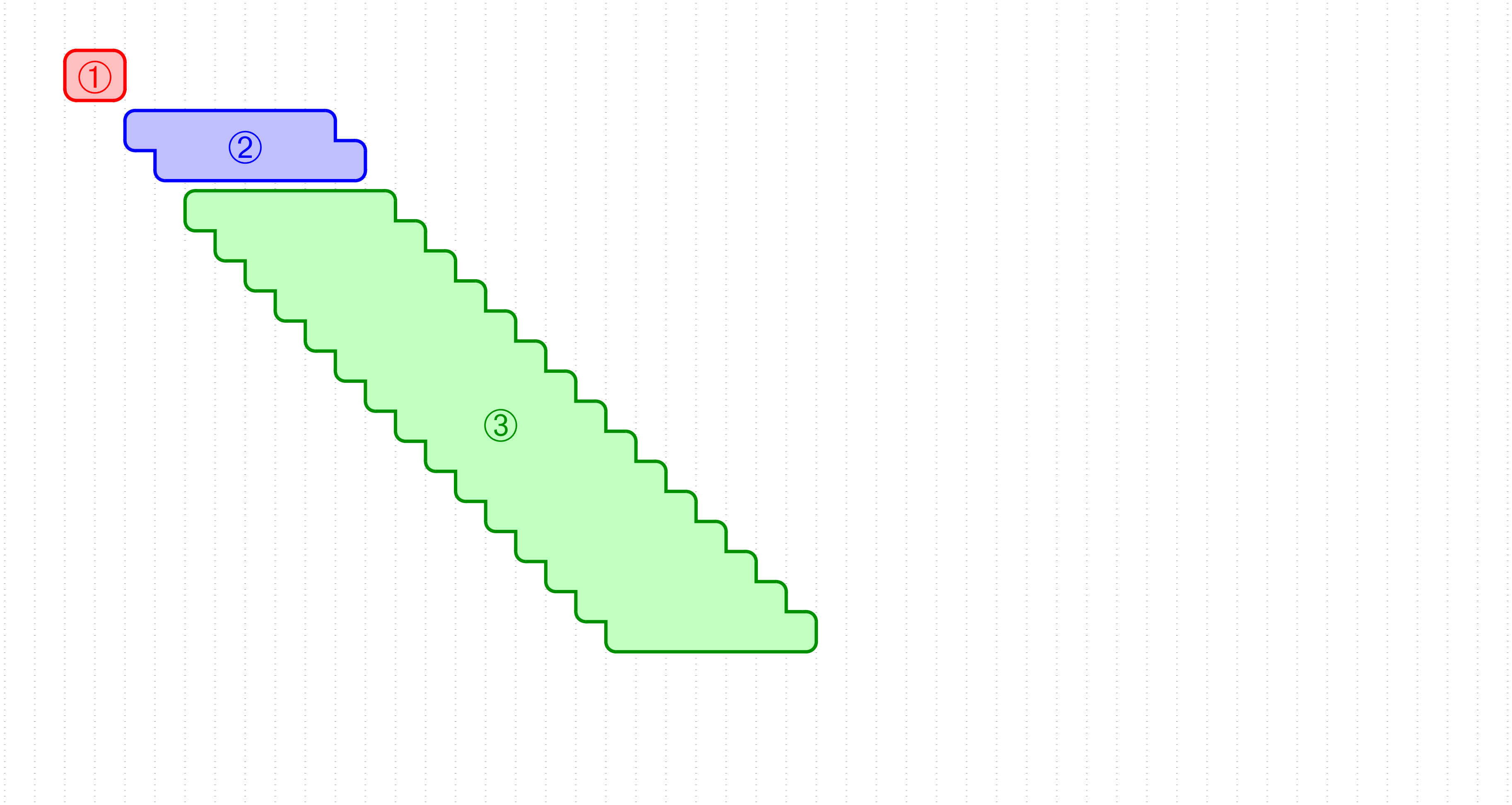
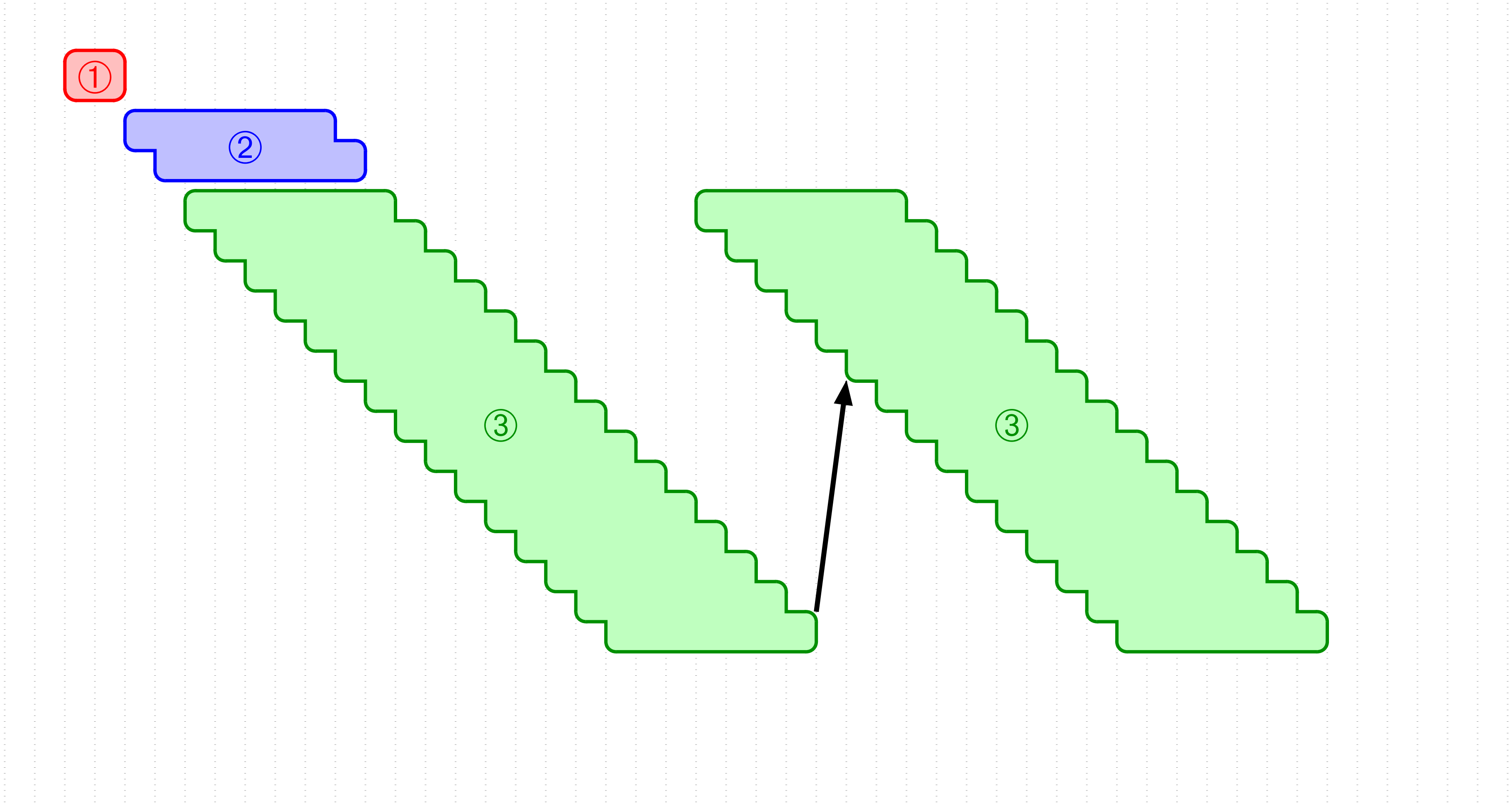


Task scheduling



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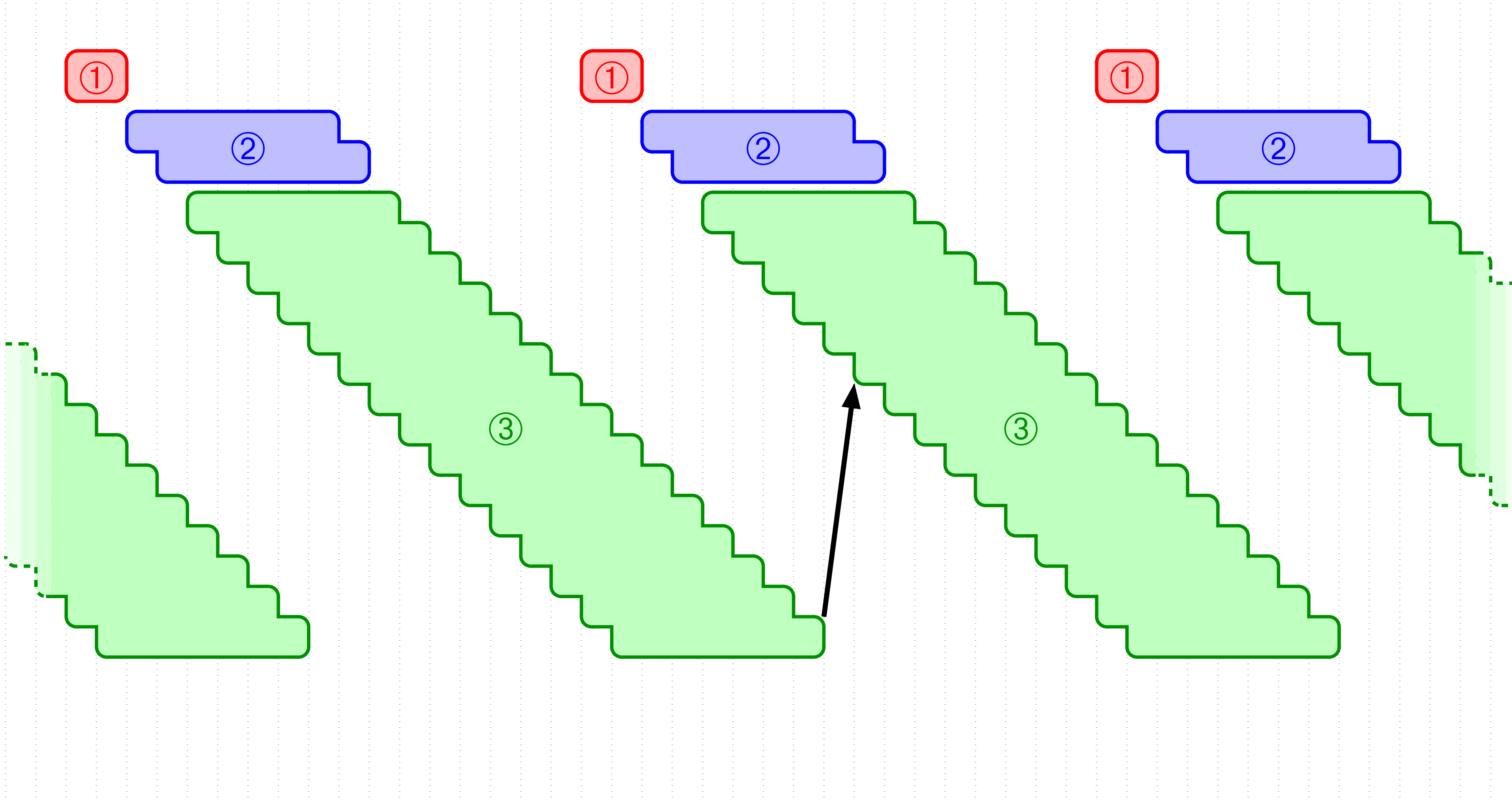
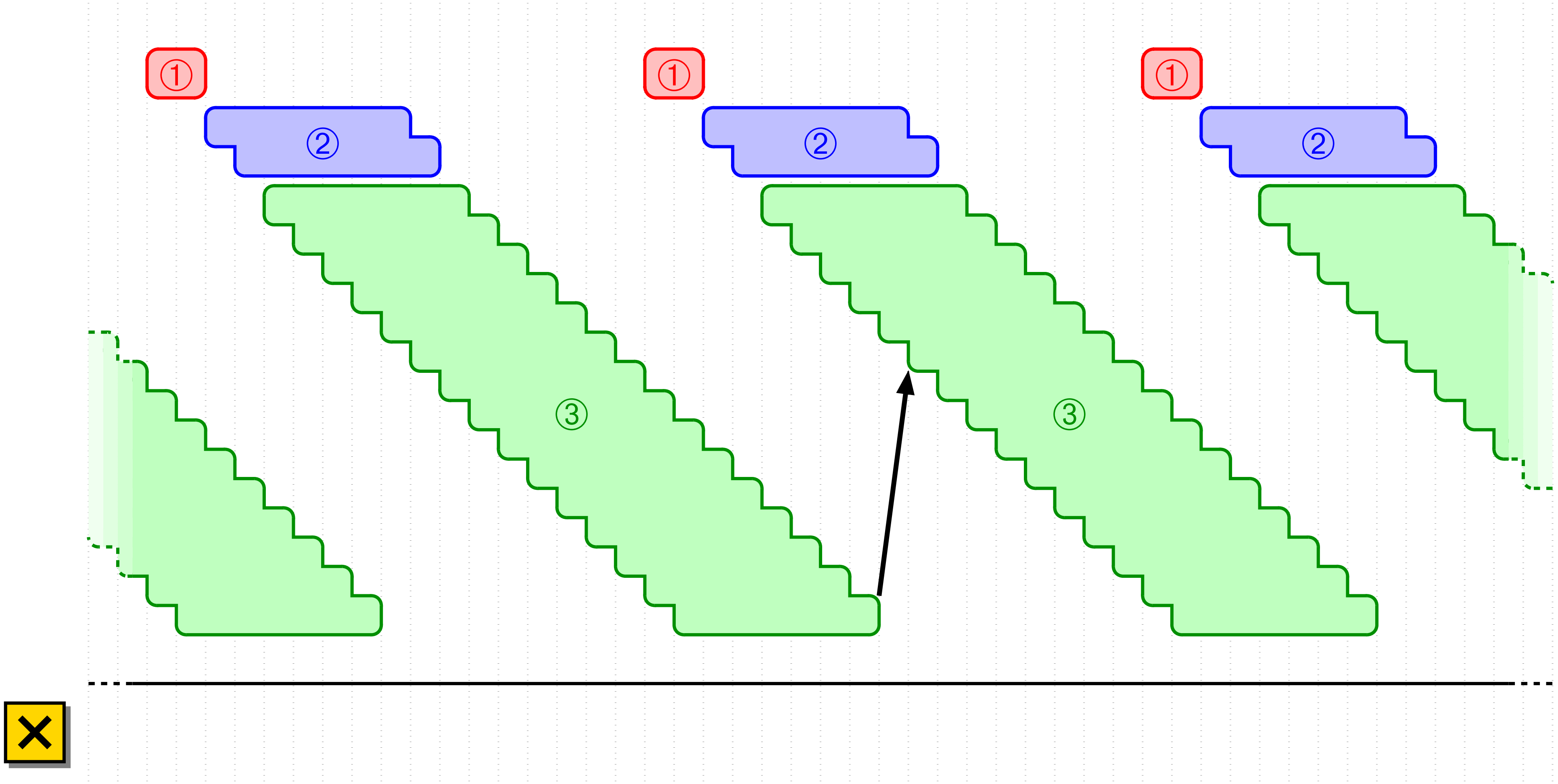


Task scheduling



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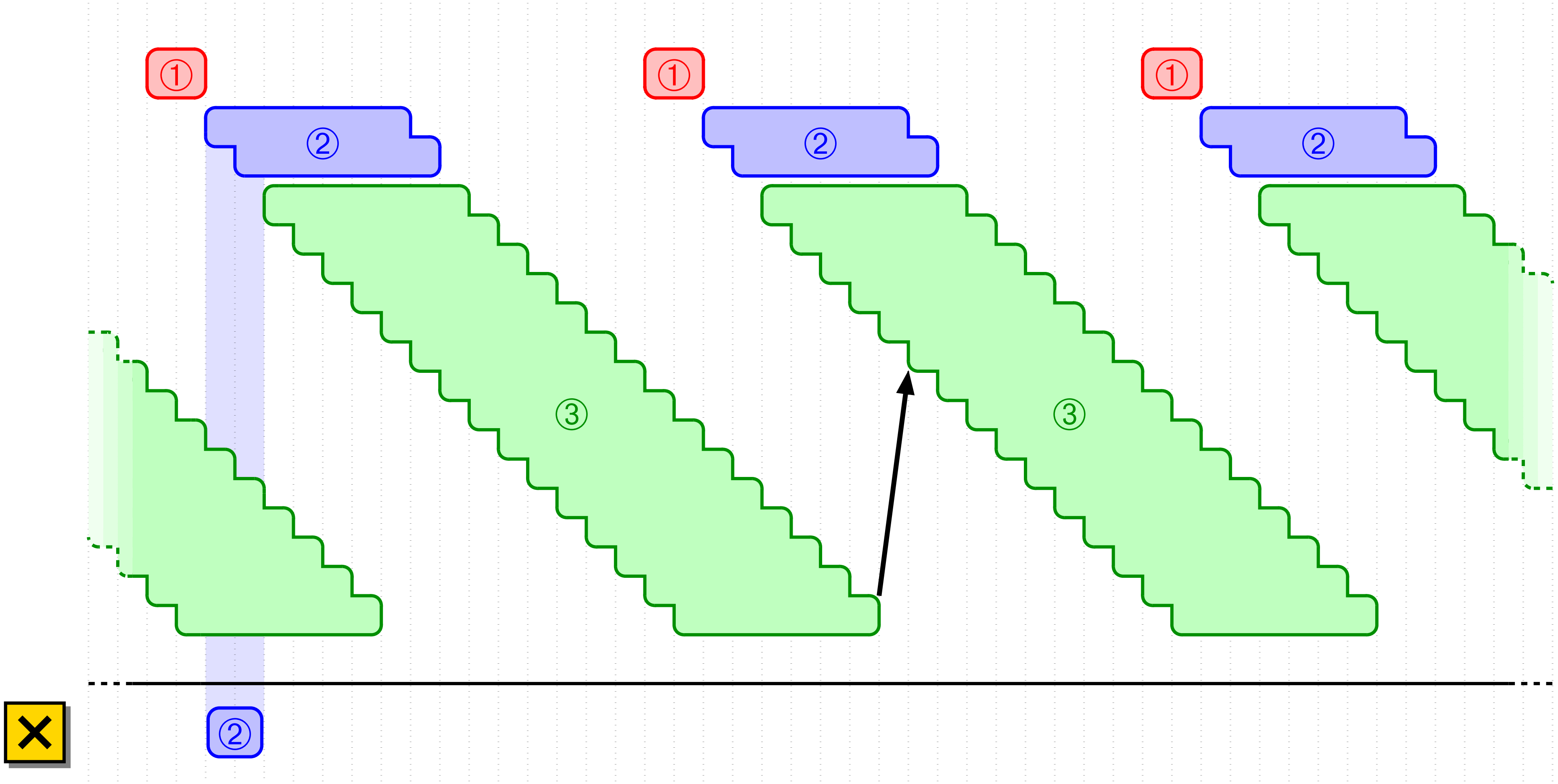

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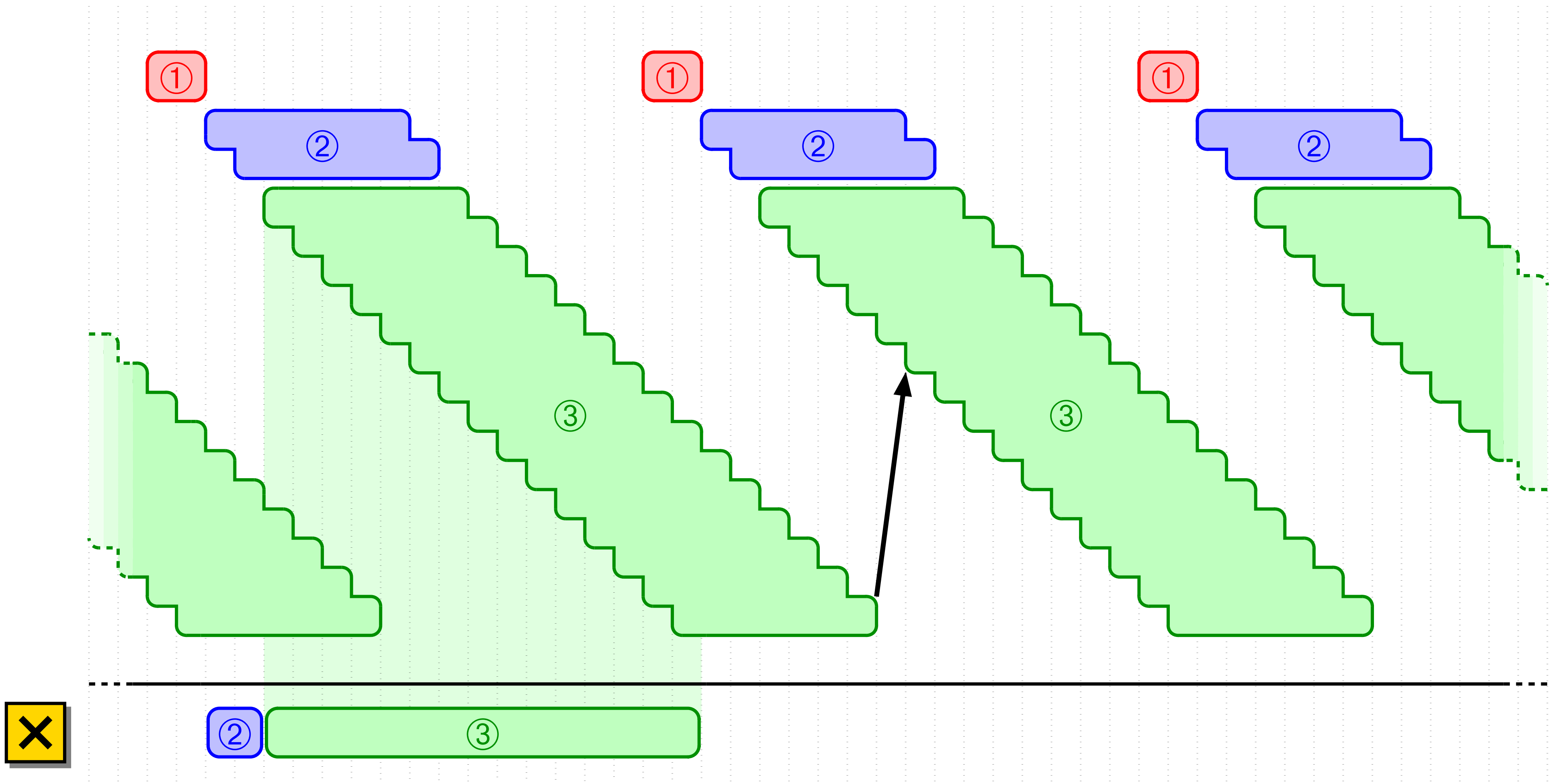

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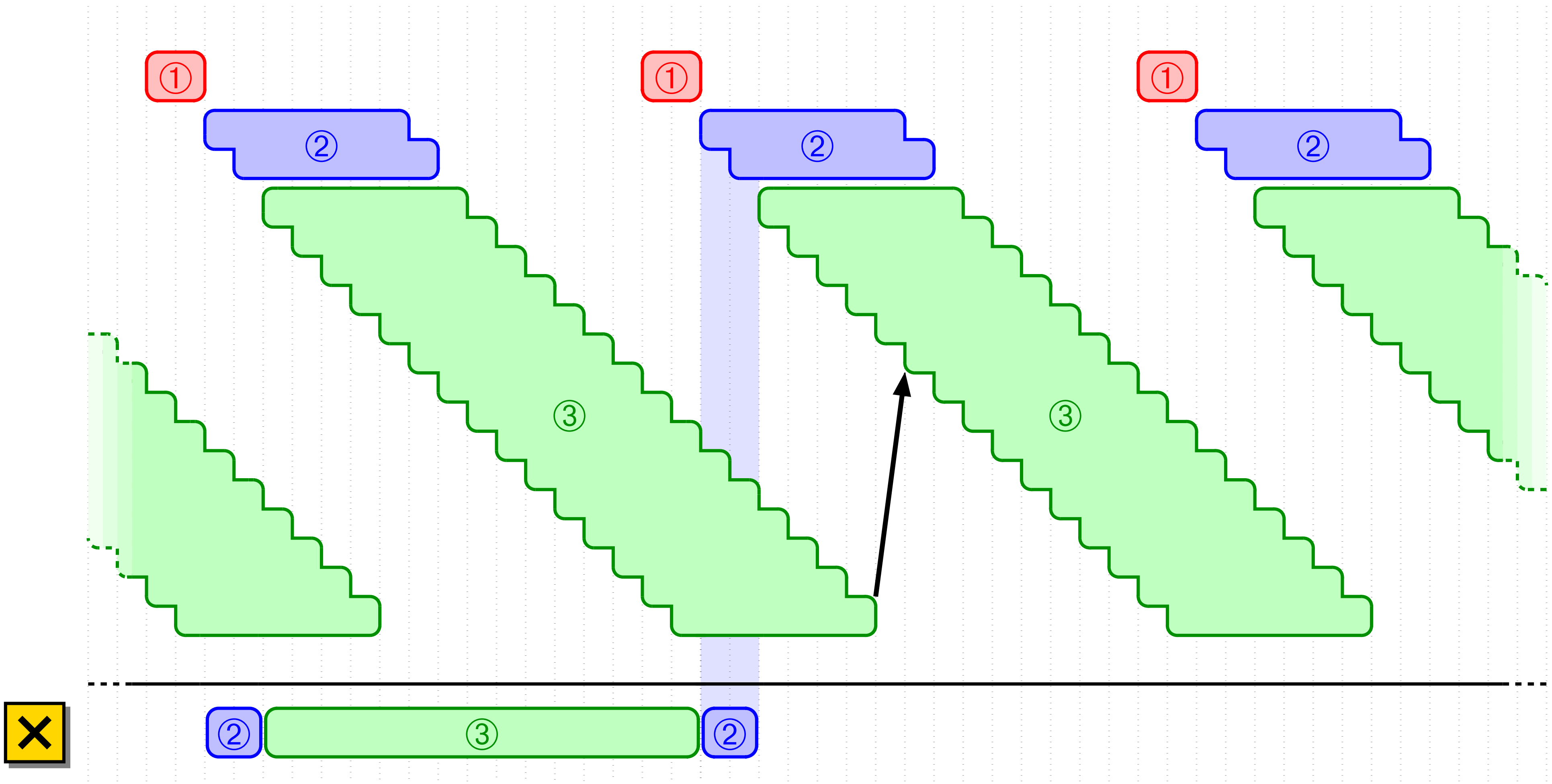
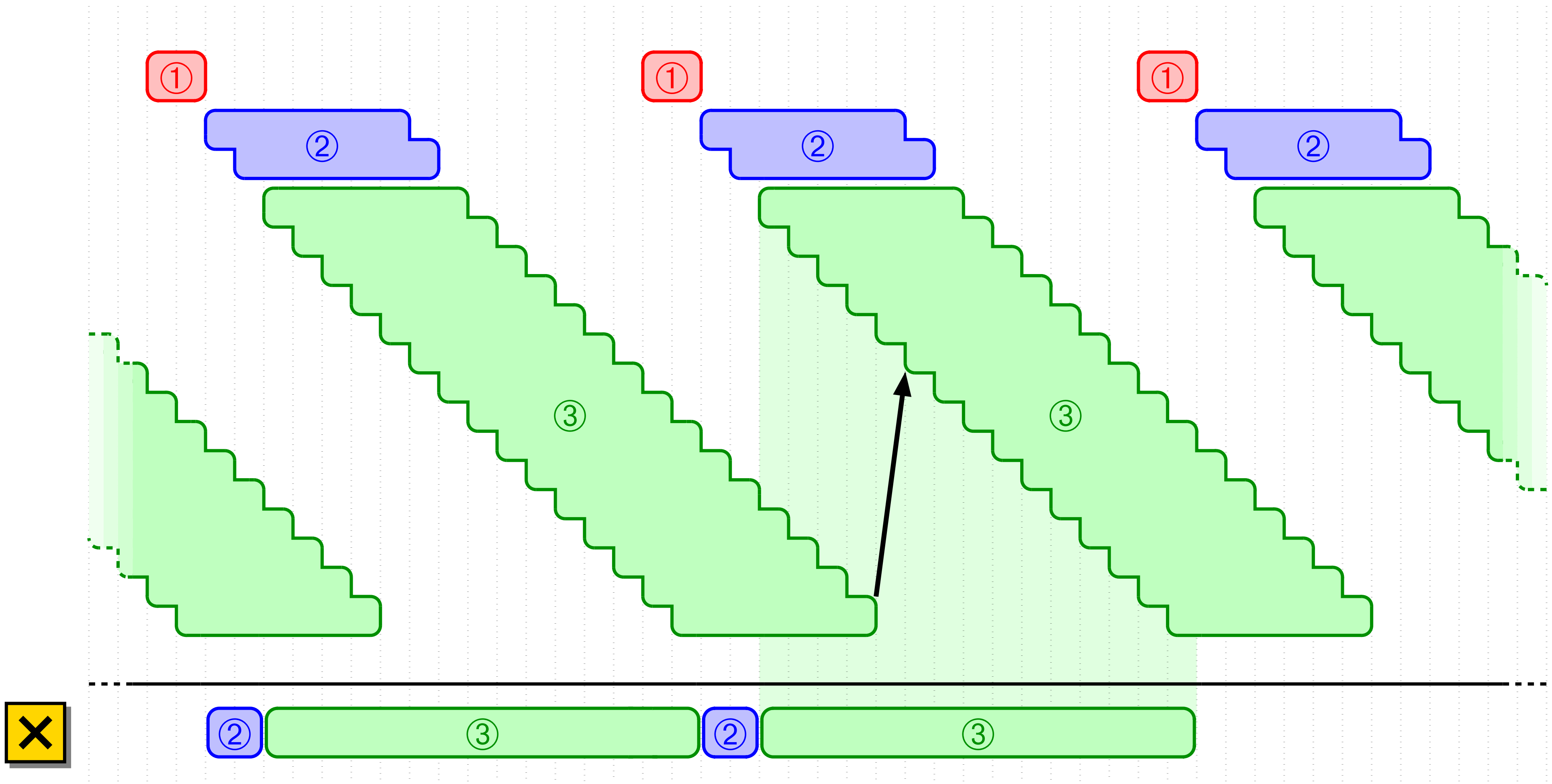
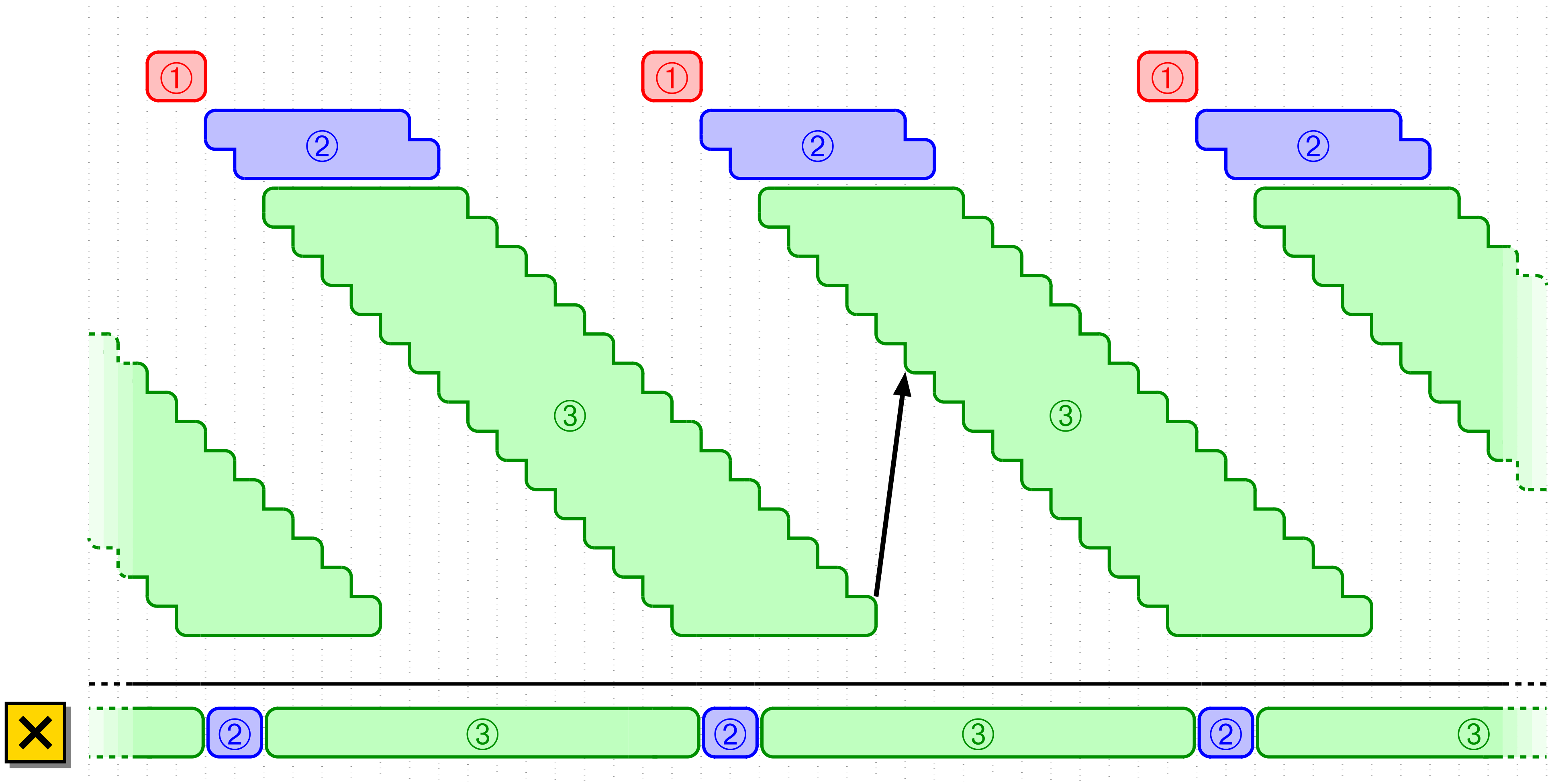


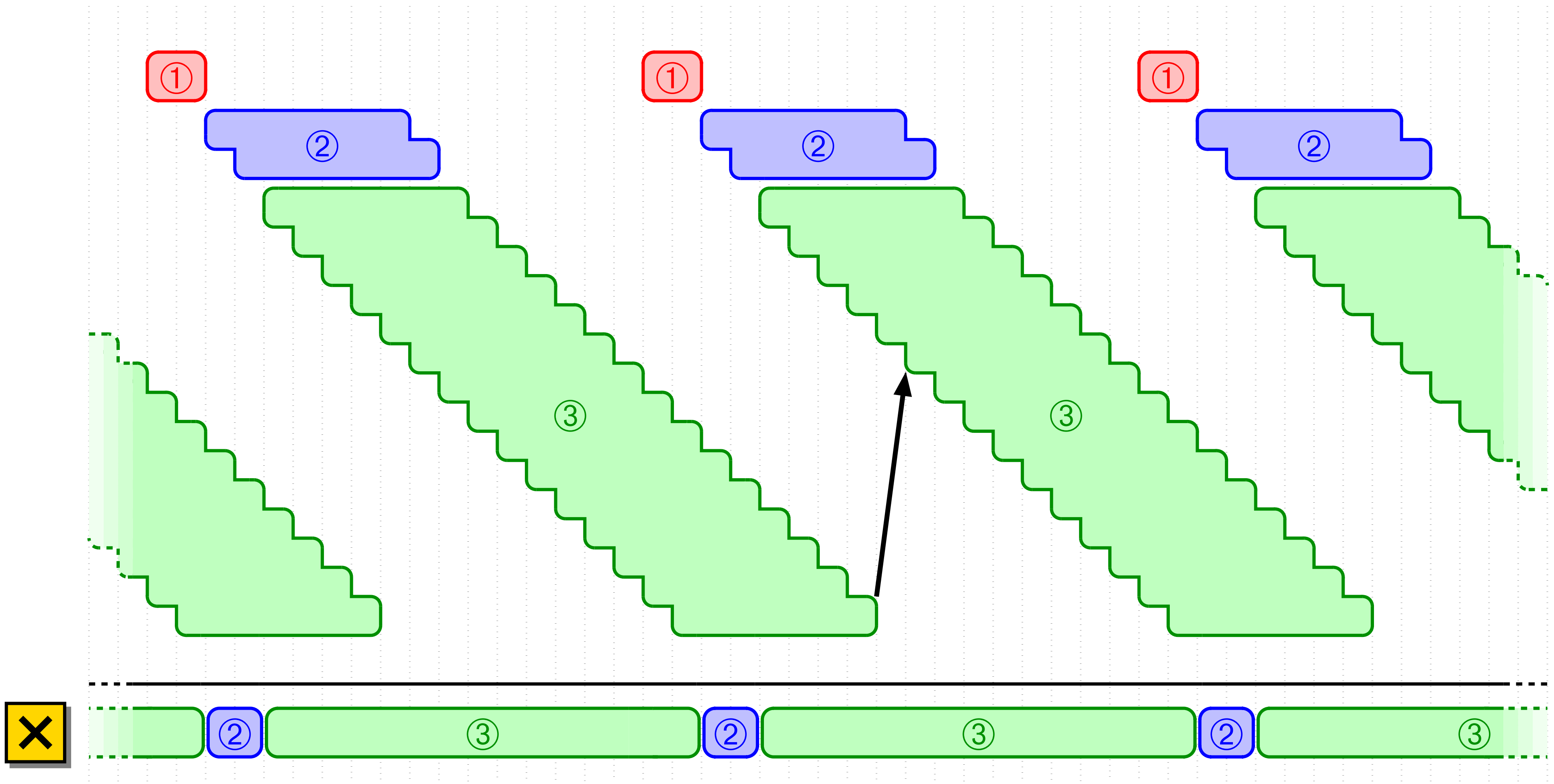
Task scheduling



Task scheduling

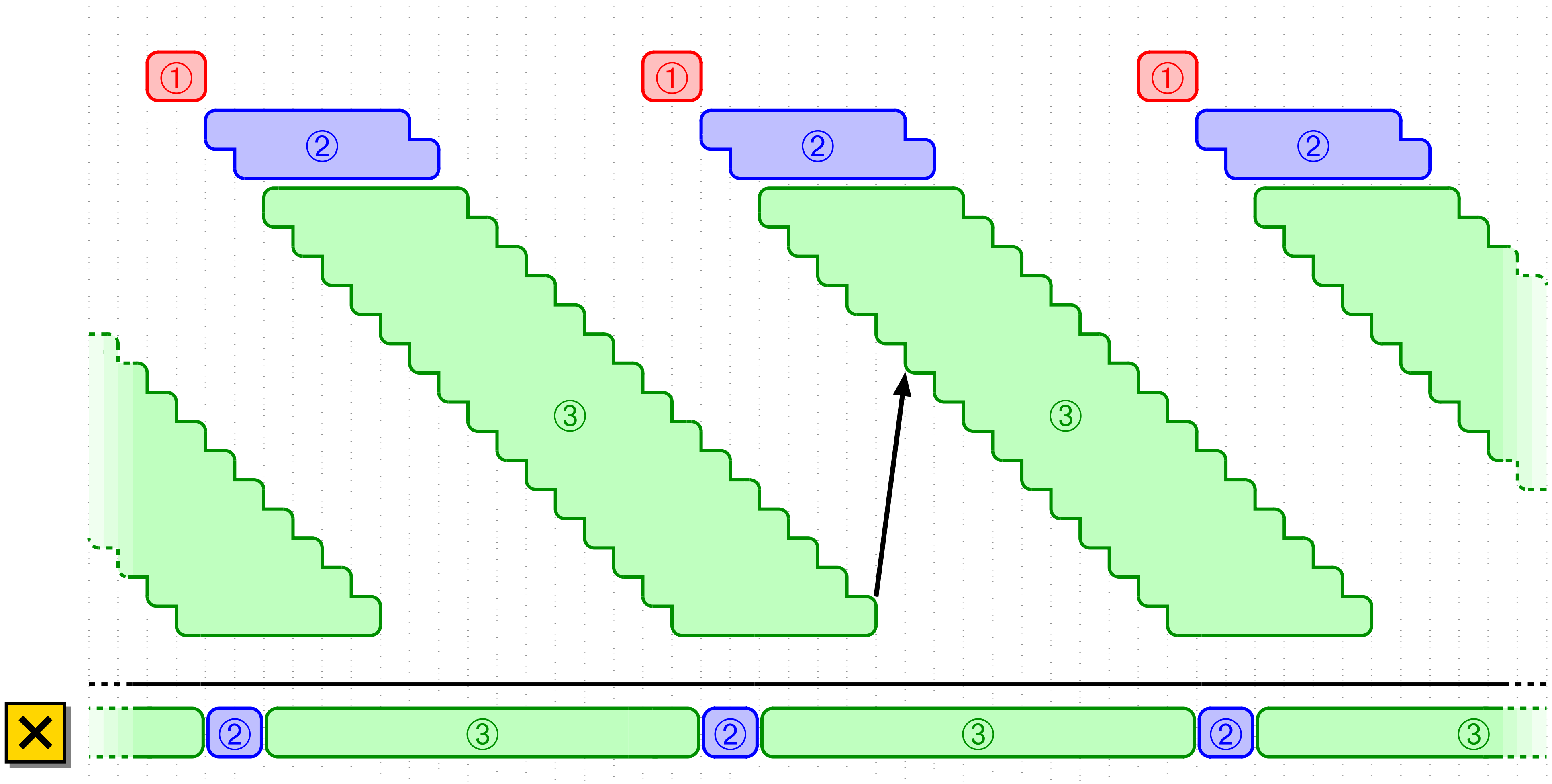


Task scheduling



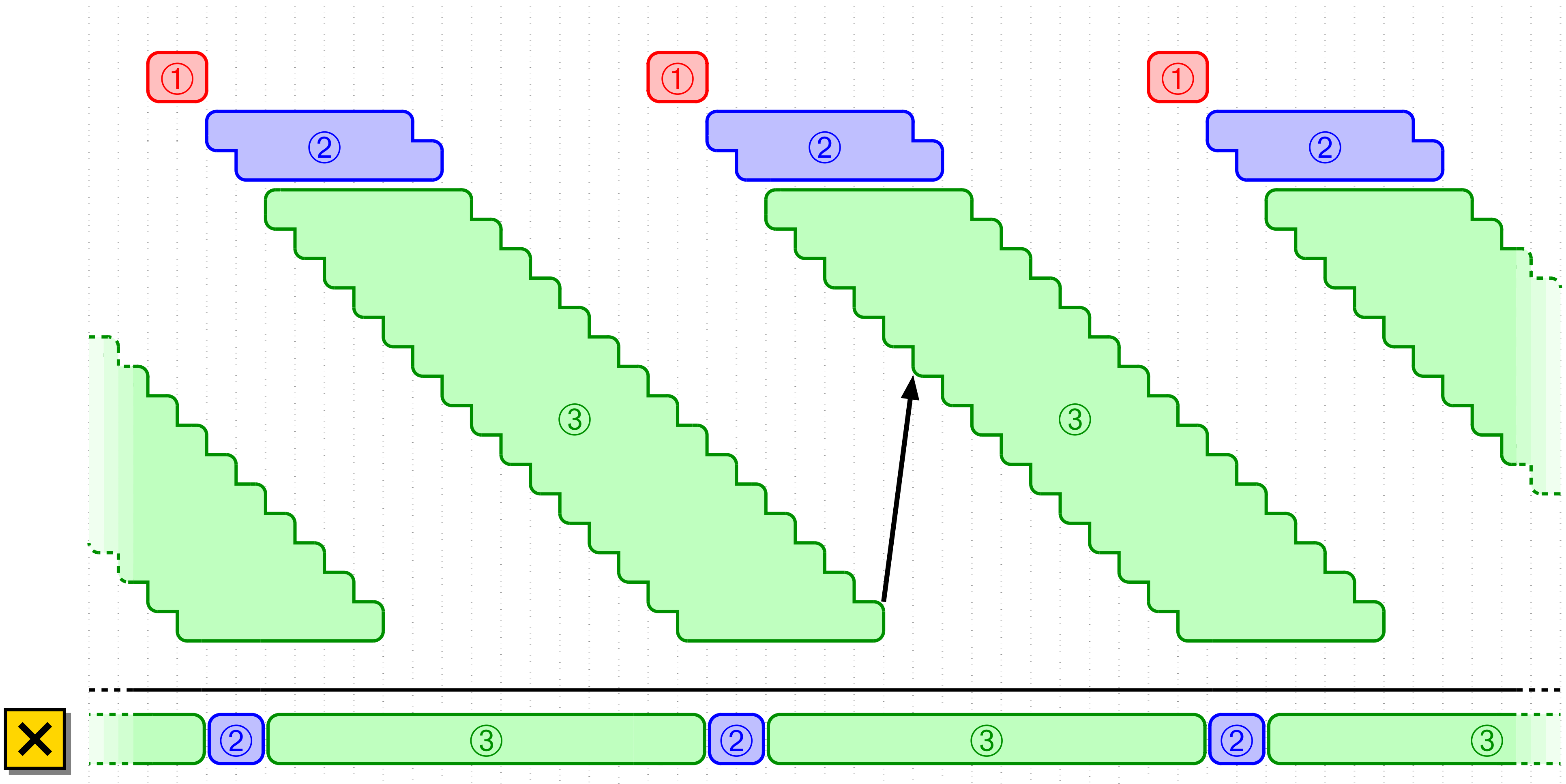
- Perfectly tight scheduling: no idle cycle

Task scheduling



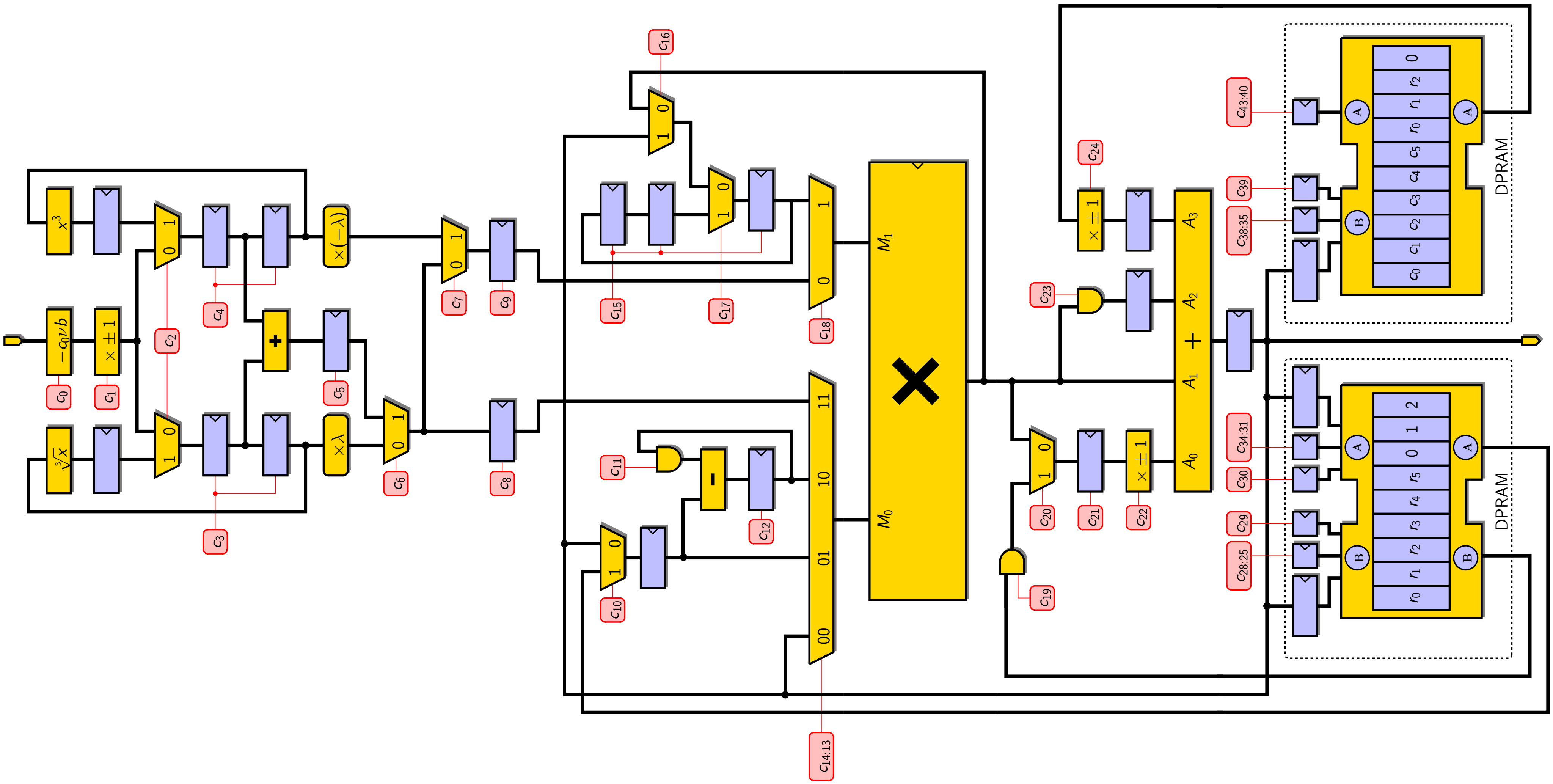
- Perfectly tight scheduling: no idle cycle
- 17 clock cycles per iteration

Task scheduling

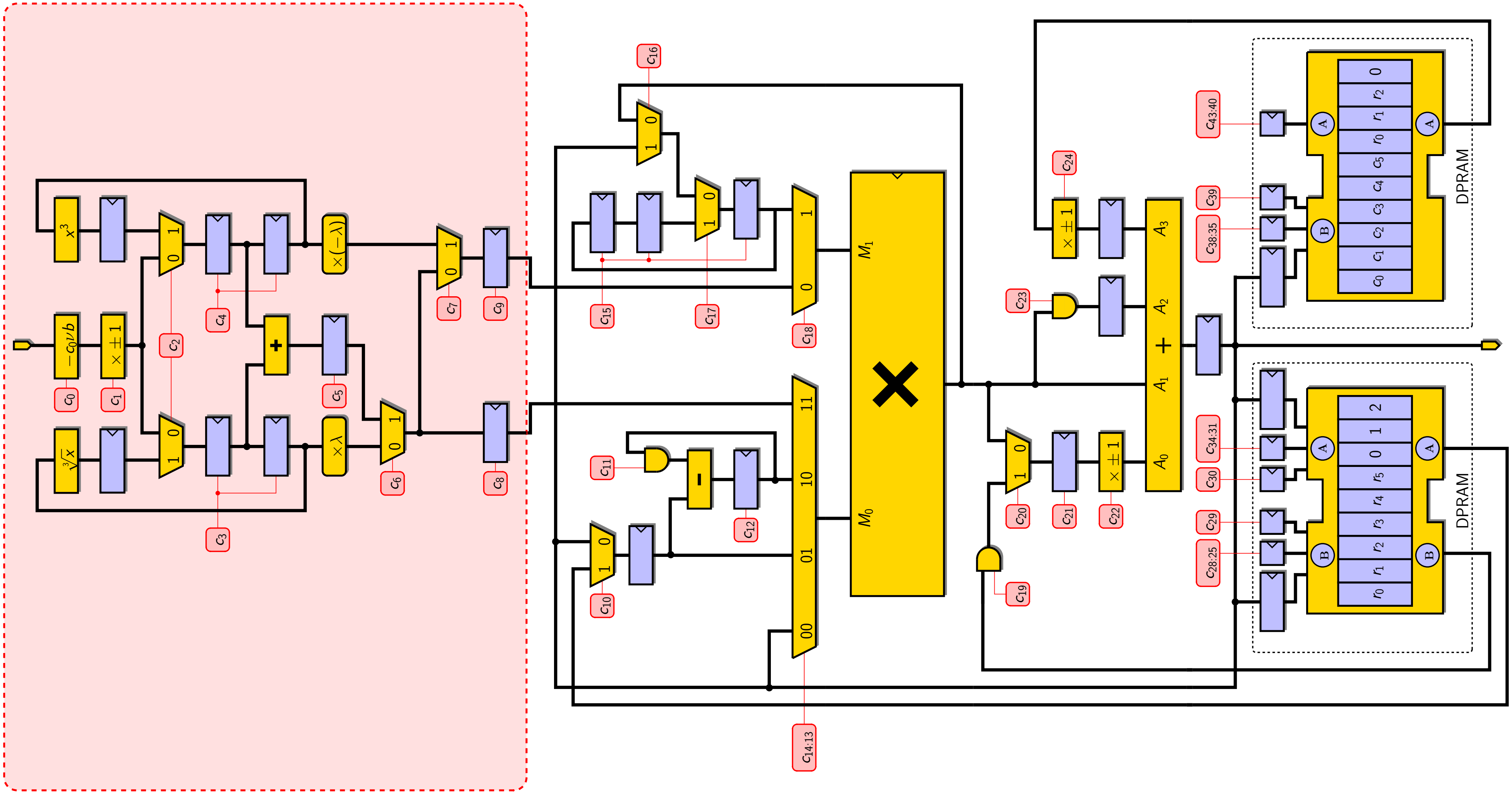


- Perfectly tight scheduling: no idle cycle
- 17 clock cycles per iteration ⇒ $17(m+1)/2$ cycles for the complete η_T pairing

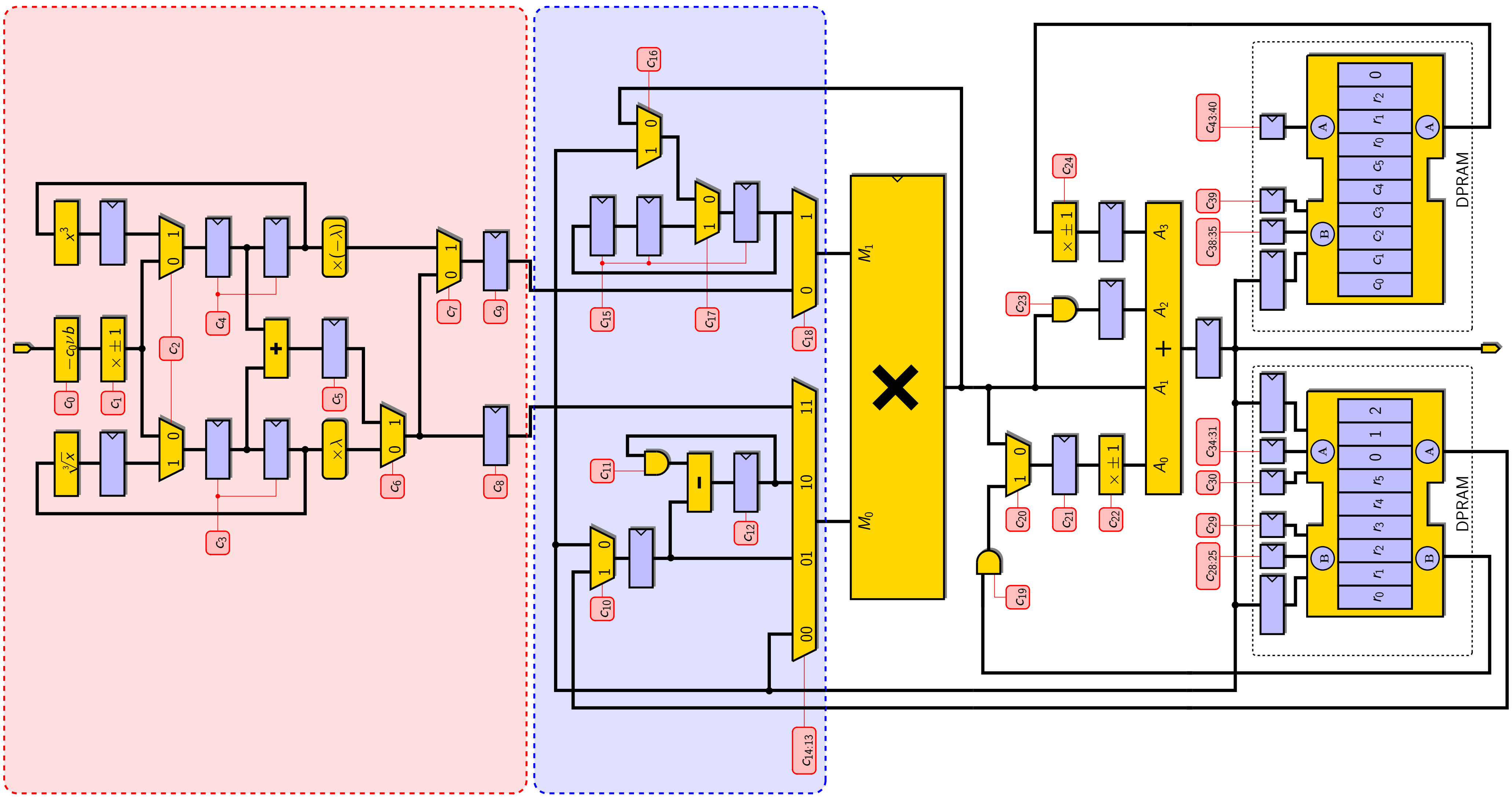
A parallel operator for the η_T pairing



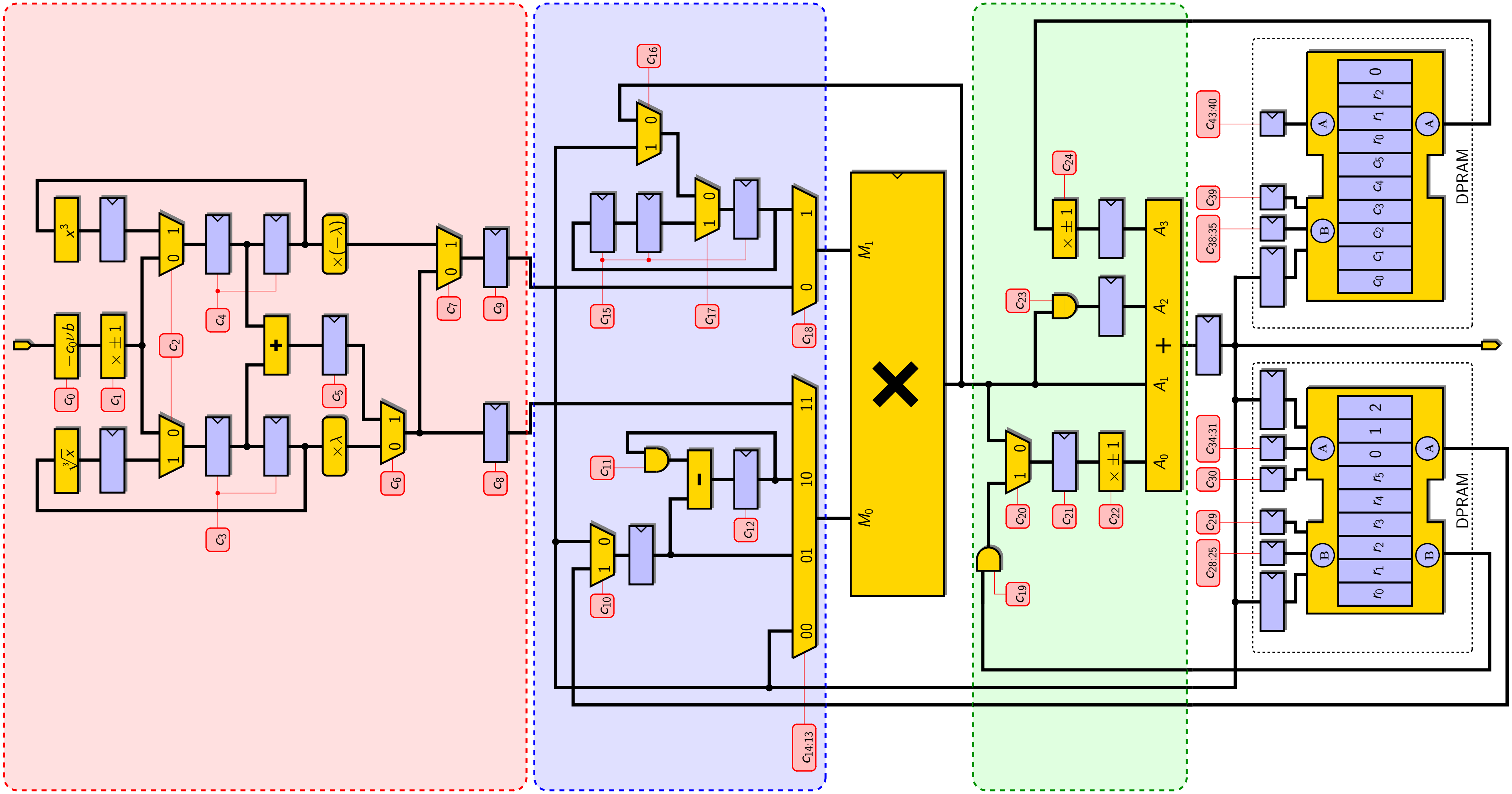
A parallel operator for the η_T pairing



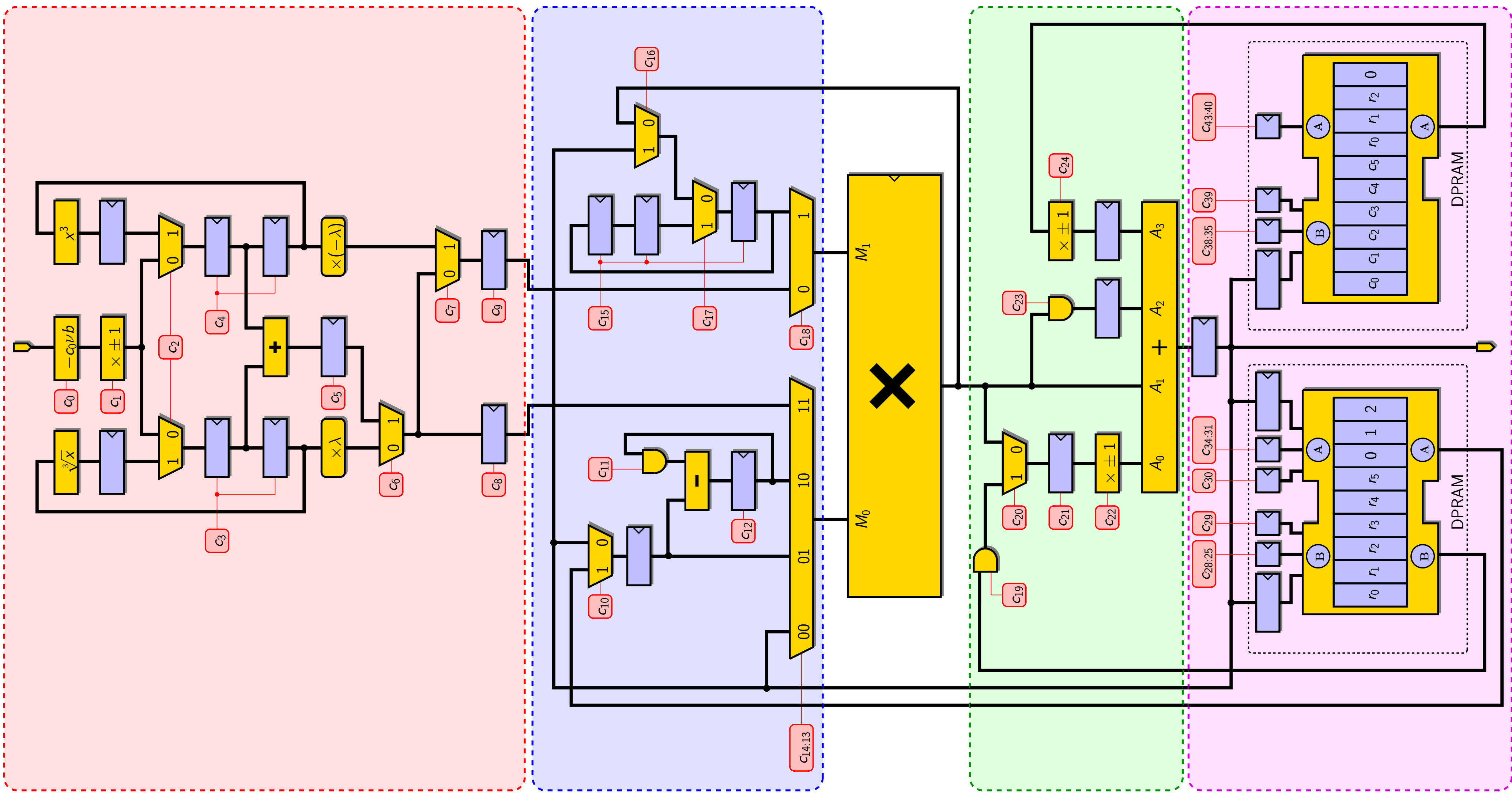
A parallel operator for the η_T pairing



A parallel operator for the η_T pairing



A parallel operator for the η_T pairing



Outline of the talk

- Previously in the Jean-Luc Beuchat Tour
- A closer look at the algorithm
- Accelerating the η_T pairing
- Accelerating the final exponentiation
- Implementation results
- Concluding thoughts

Outline of the talk

- Previously in the Jean-Luc Beuchat Tour
- A closer look at the algorithm
- Accelerating the η_T pairing
- Accelerating the final exponentiation (in characteristic 3)
- Implementation results
- Concluding thoughts

The final exponentiation

- Compute $\hat{e}(P, Q)$ as $\eta_T(P, Q)^M$ with $\eta_T(P, Q) \in \mathbb{F}_{3^{6m}}^{\times}$ and

$$M = \left(3^{3m} - 1\right)\left(3^m + 1\right)\left(3^m + 1 \mp 3^{(m+1)/2}\right)$$

The final exponentiation

- Compute $\hat{e}(P, Q)$ as $\eta_T(P, Q)^M$ with $\eta_T(P, Q) \in \mathbb{F}_{3^{6m}}^{\times}$ and

$$M = \left(3^{3m} - 1\right)\left(3^m + 1\right)\left(3^m + 1 \mp 3^{(m+1)/2}\right)$$

- Operations over $\mathbb{F}_{3^m}$: $73\,\times$, $3m + 3$ Frobenius, $3m + 175\,+$, and 1 inversion

The final exponentiation

- Compute $\hat{e}(P, Q)$ as $\eta_T(P, Q)^M$ with $\eta_T(P, Q) \in \mathbb{F}_{3^{6m}}^{\times}$ and

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- The final exponentiation is much cheaper than the η_T pairing

- Challenge for the final exponentiation:
  - computation in the same time as the η_T pairing
  - ... using as few resources as possible

The final exponentiation

- First idea: use the unified operator
  - the smallest architecture supporting all the required operations over $\mathbb{F}_{3^m}$
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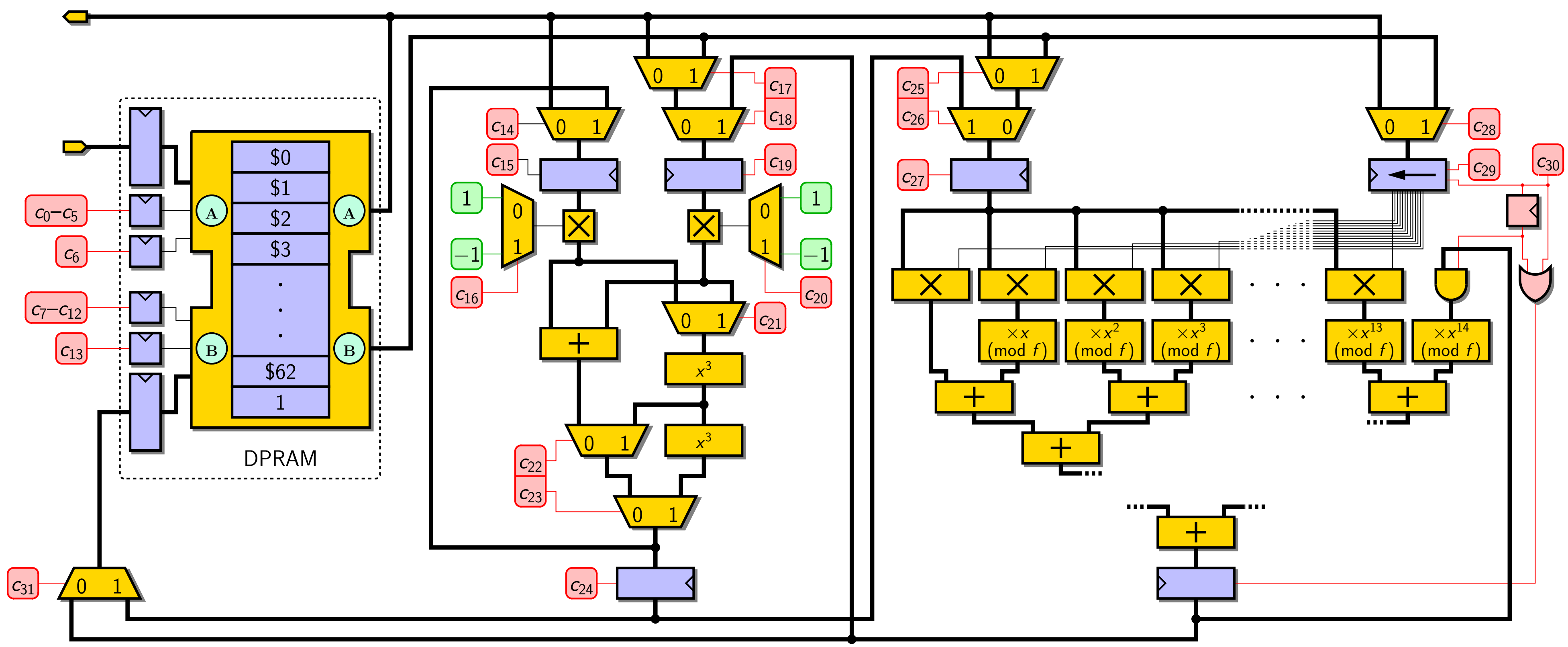
- New coprocessor with two arithmetic units:
  - a standalone multiplier, based on a parallel-serial scheme
  - a unified operator supporting addition/subtraction, Frobenius map and double Frobenius map

A coprocessor for the final exponentiation



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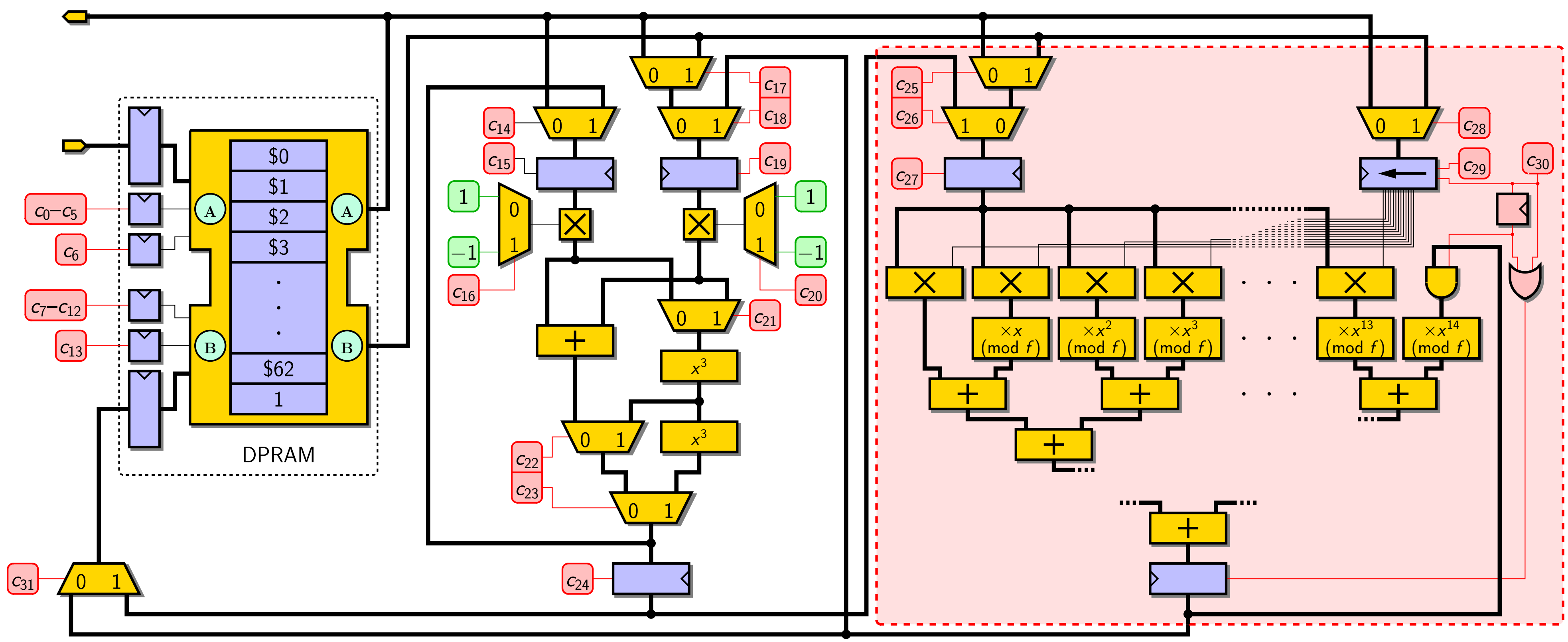

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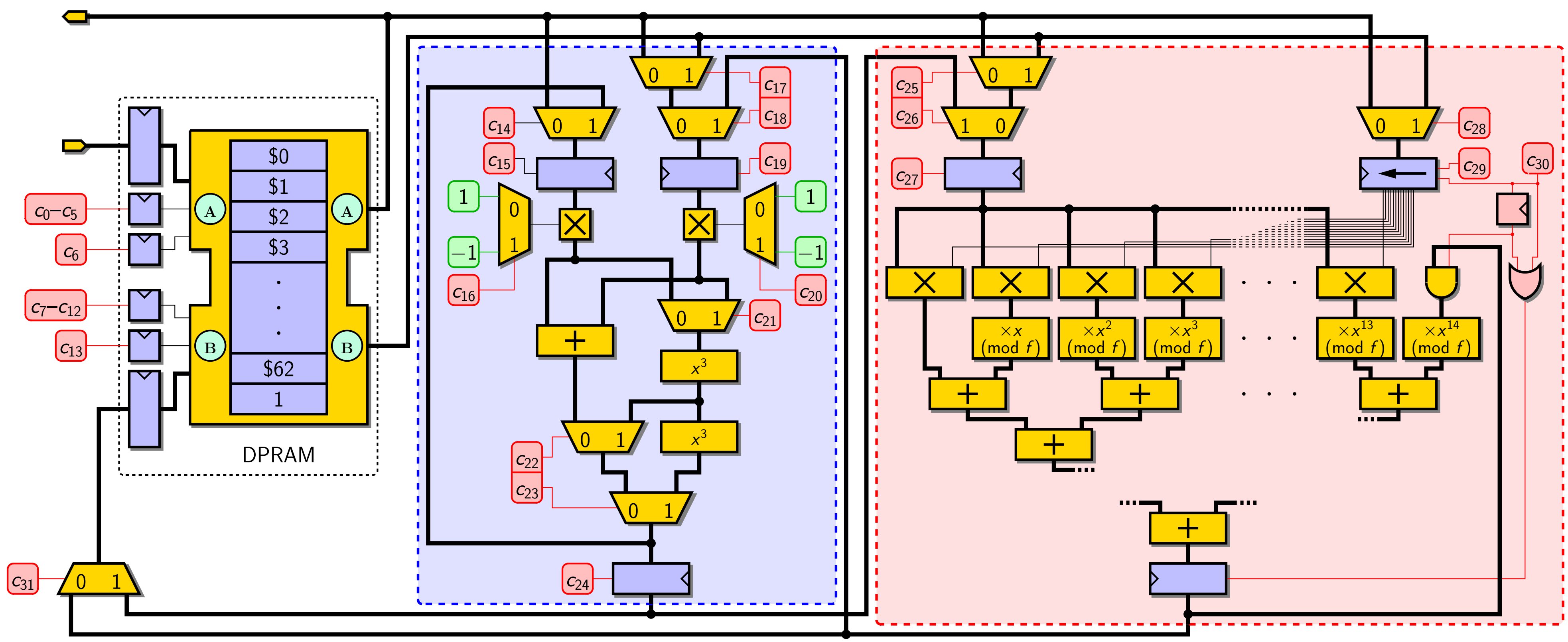

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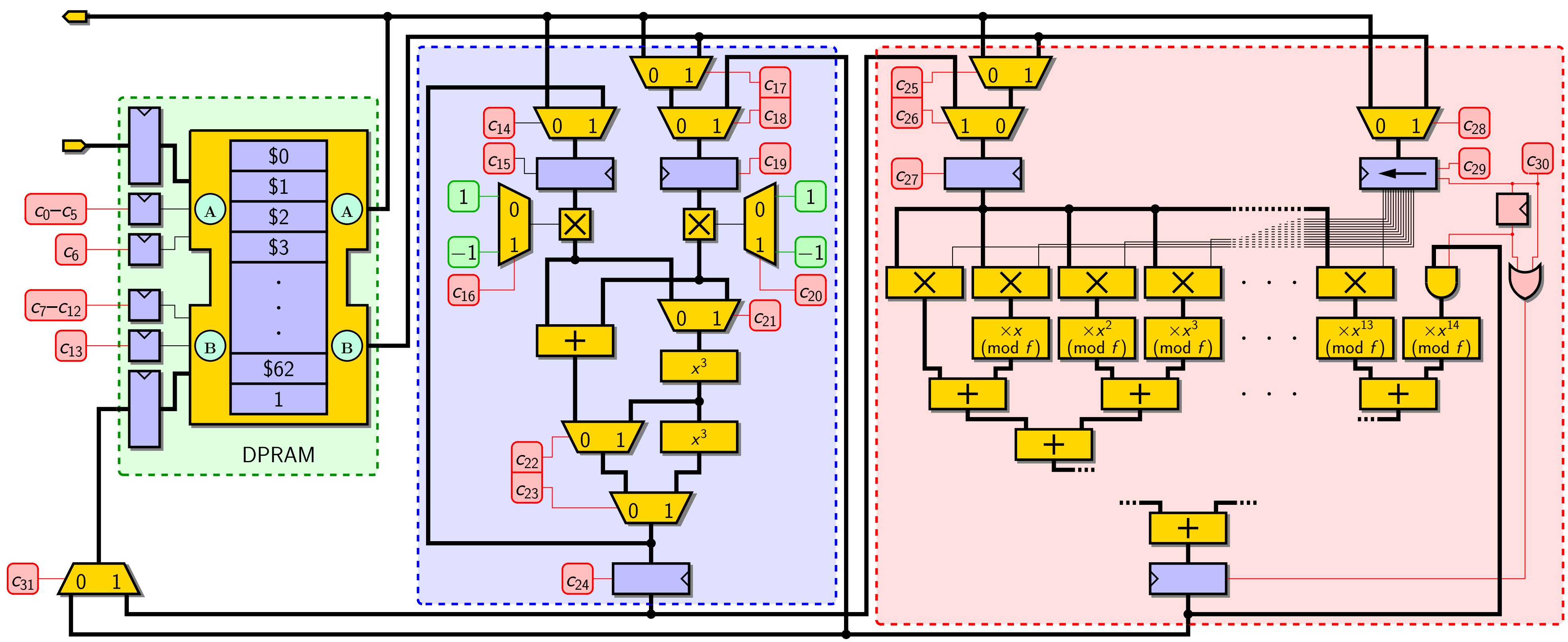


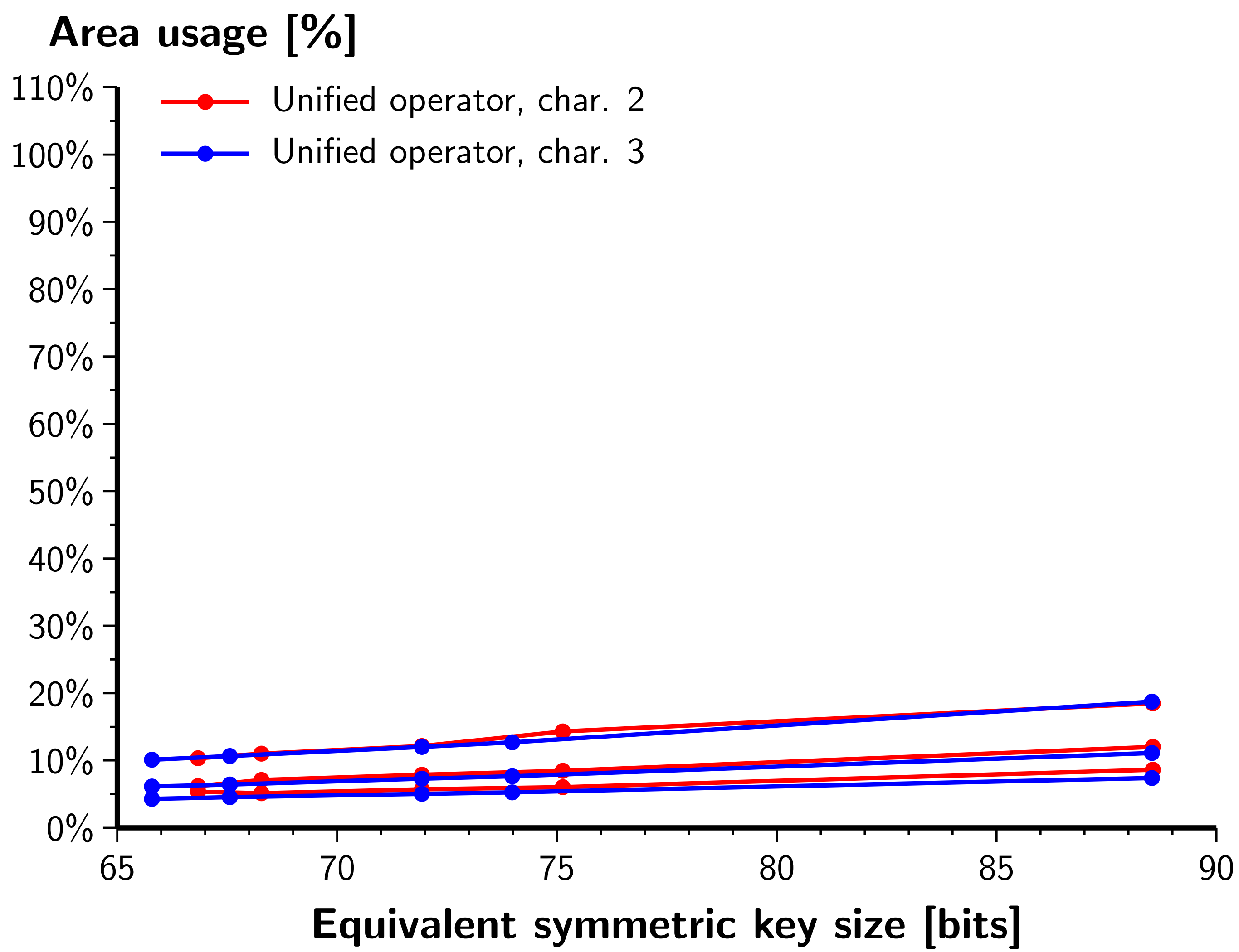
Outline of the talk

- Previously in the Jean-Luc Beuchat Tour
- A closer look at the algorithm
- Accelerating the η_T pairing
- Accelerating the final exponentiation
- Implementation results
- Concluding thoughts

Experimental setup

- Full coprocessor for computation of the Tate pairing
- Architecture based on the two parallel accelerators
- Prototyped on a Xilinx Virtex-II Pro 50 FPGA (larger model)
- Post place-and-route results: area, computation time, AT product

Coprocessor area



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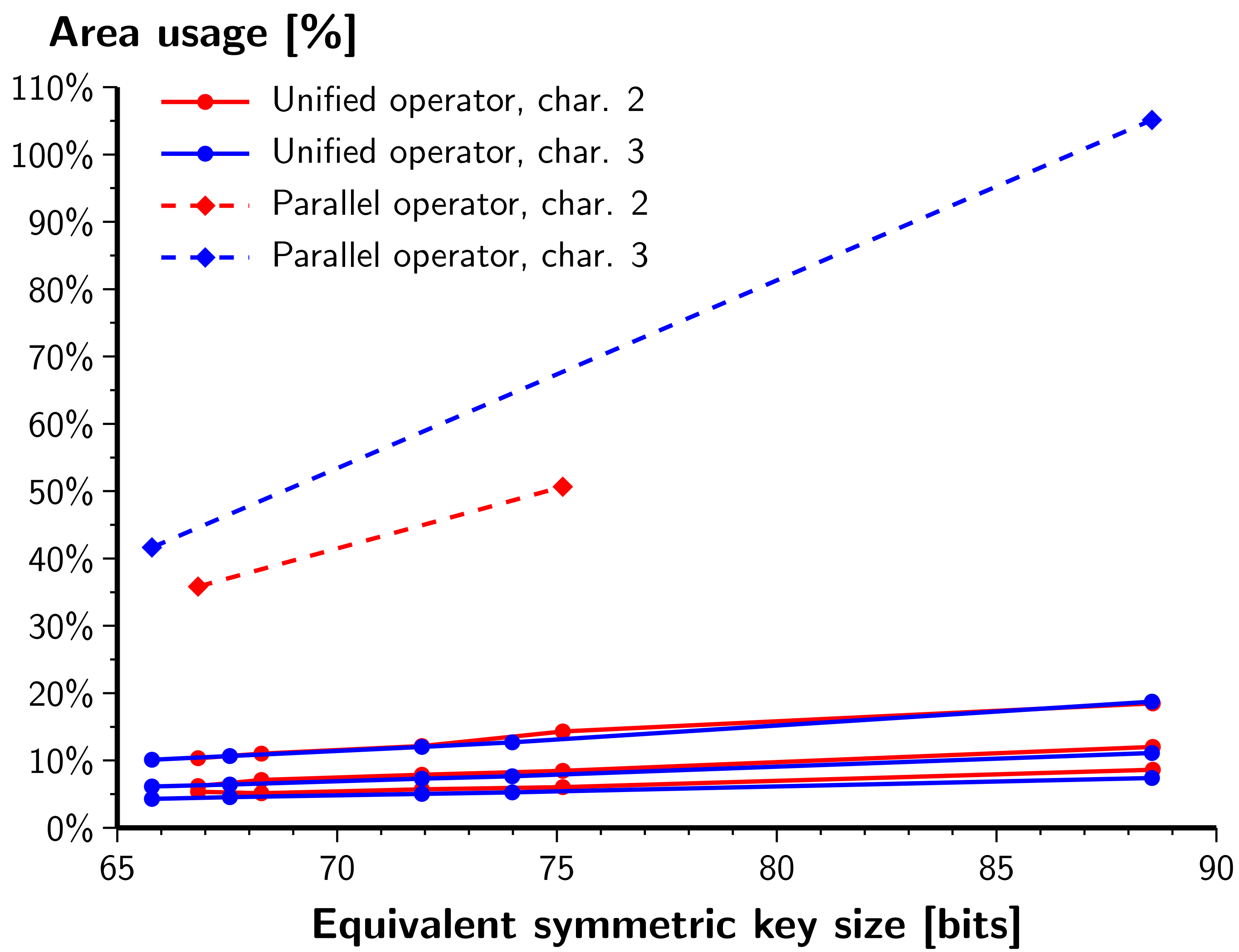
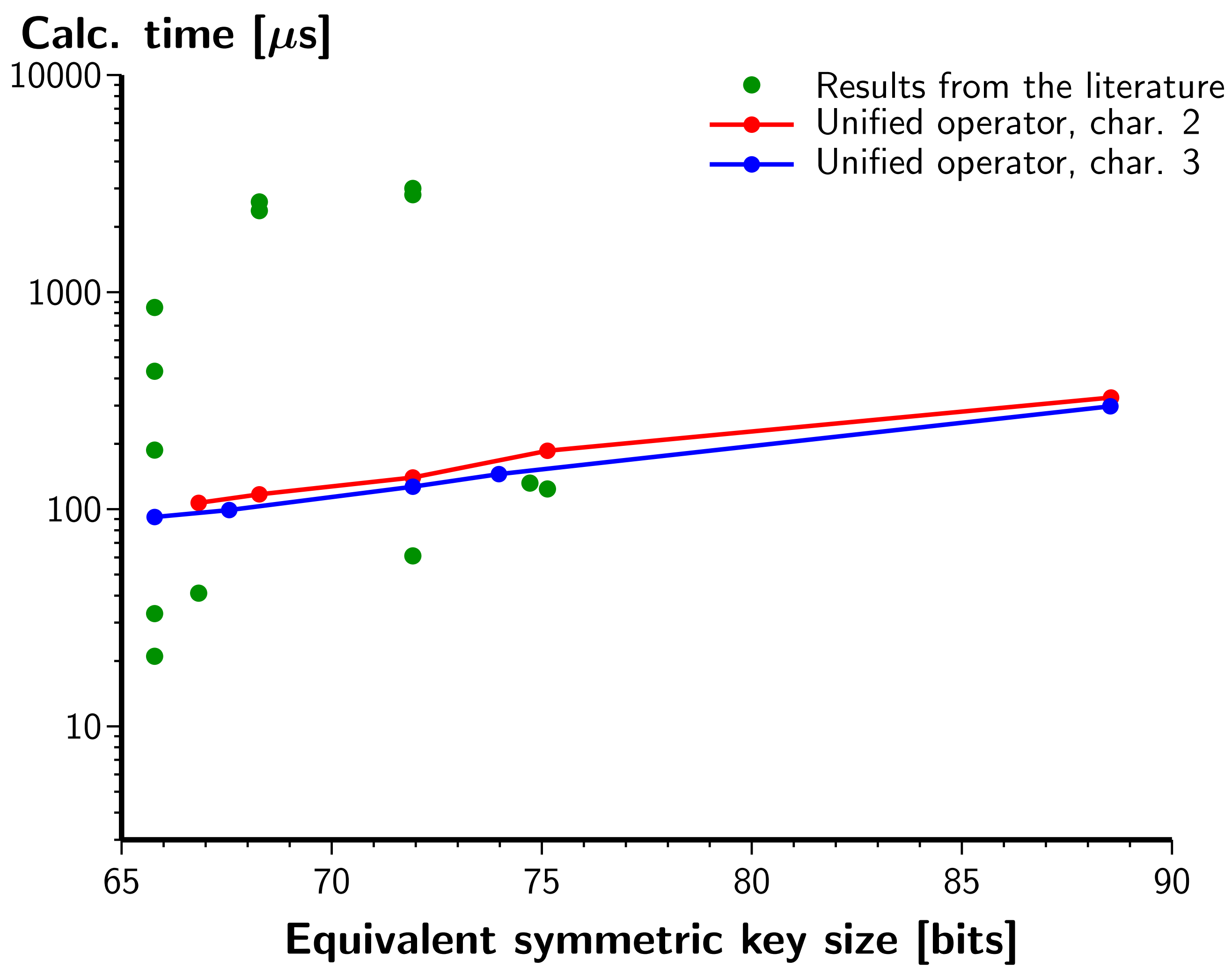


Calculation time



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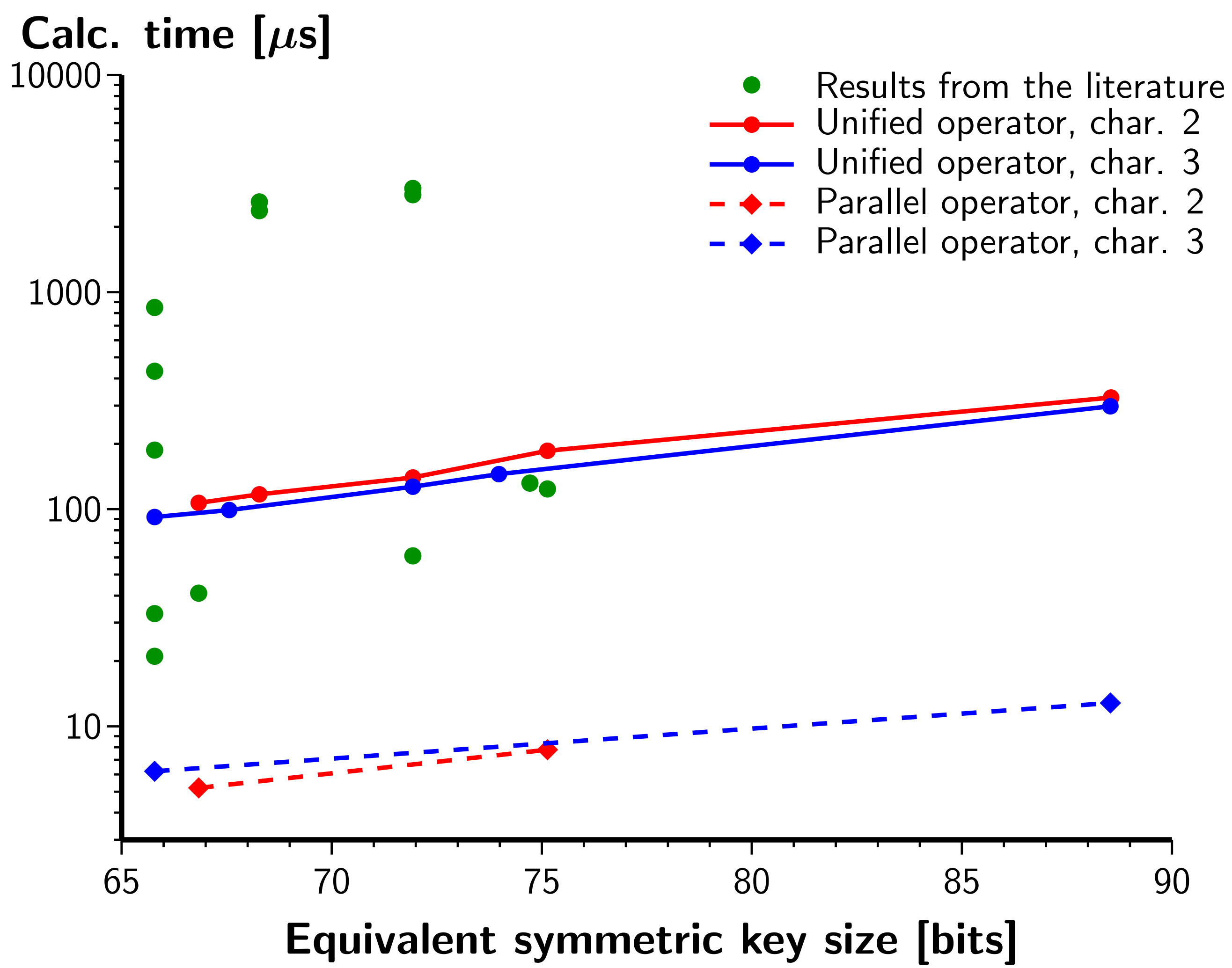
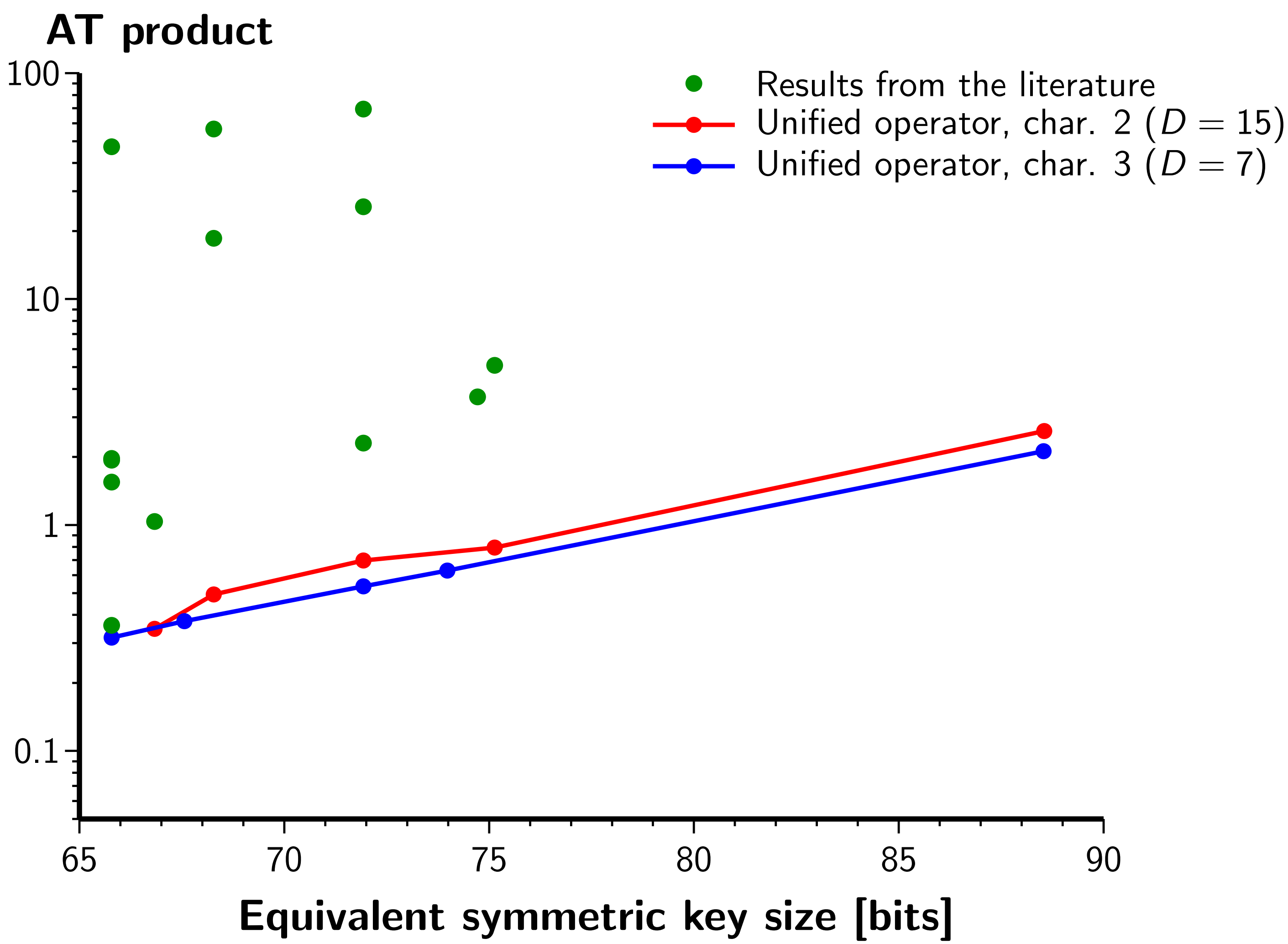
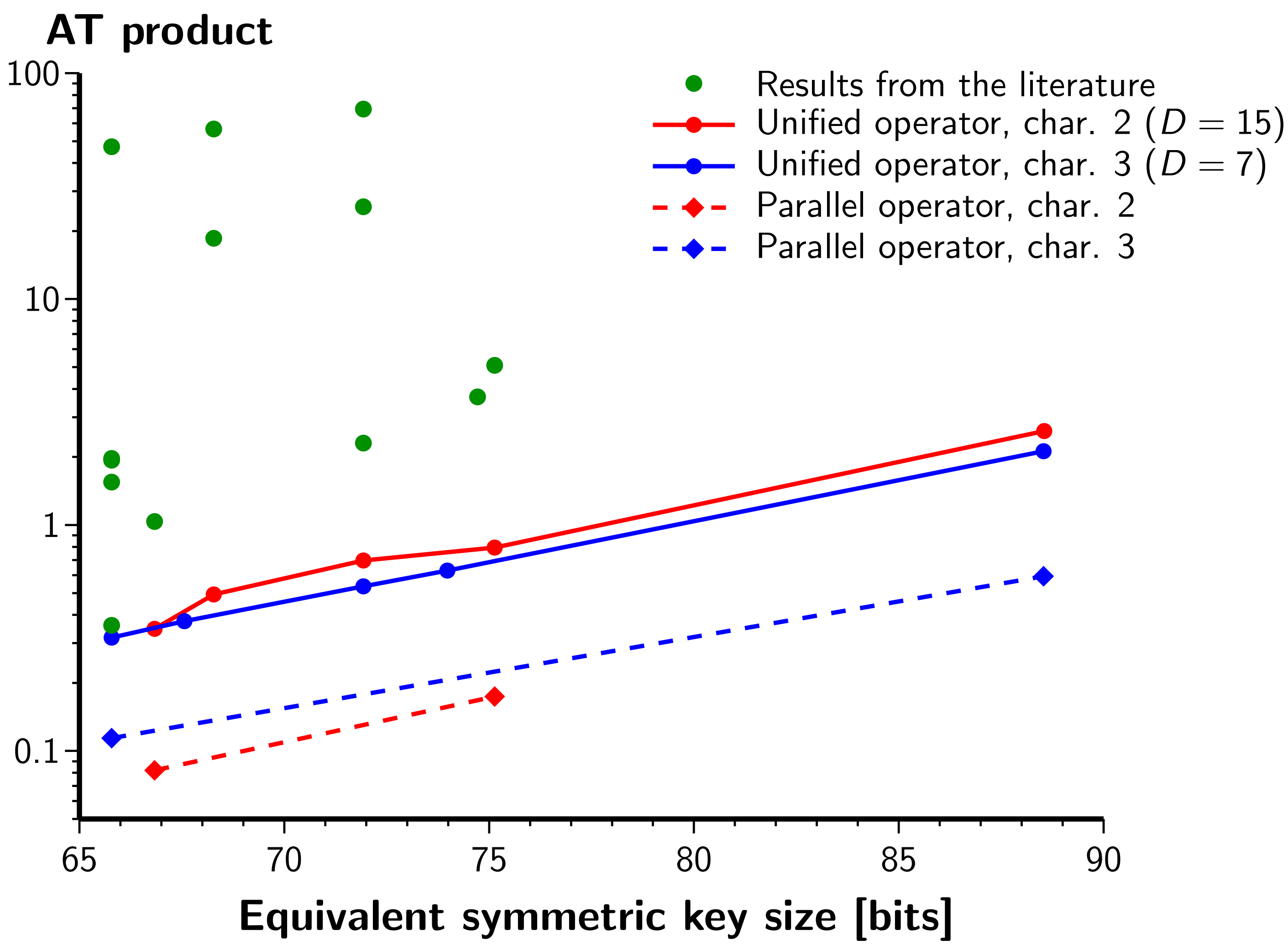


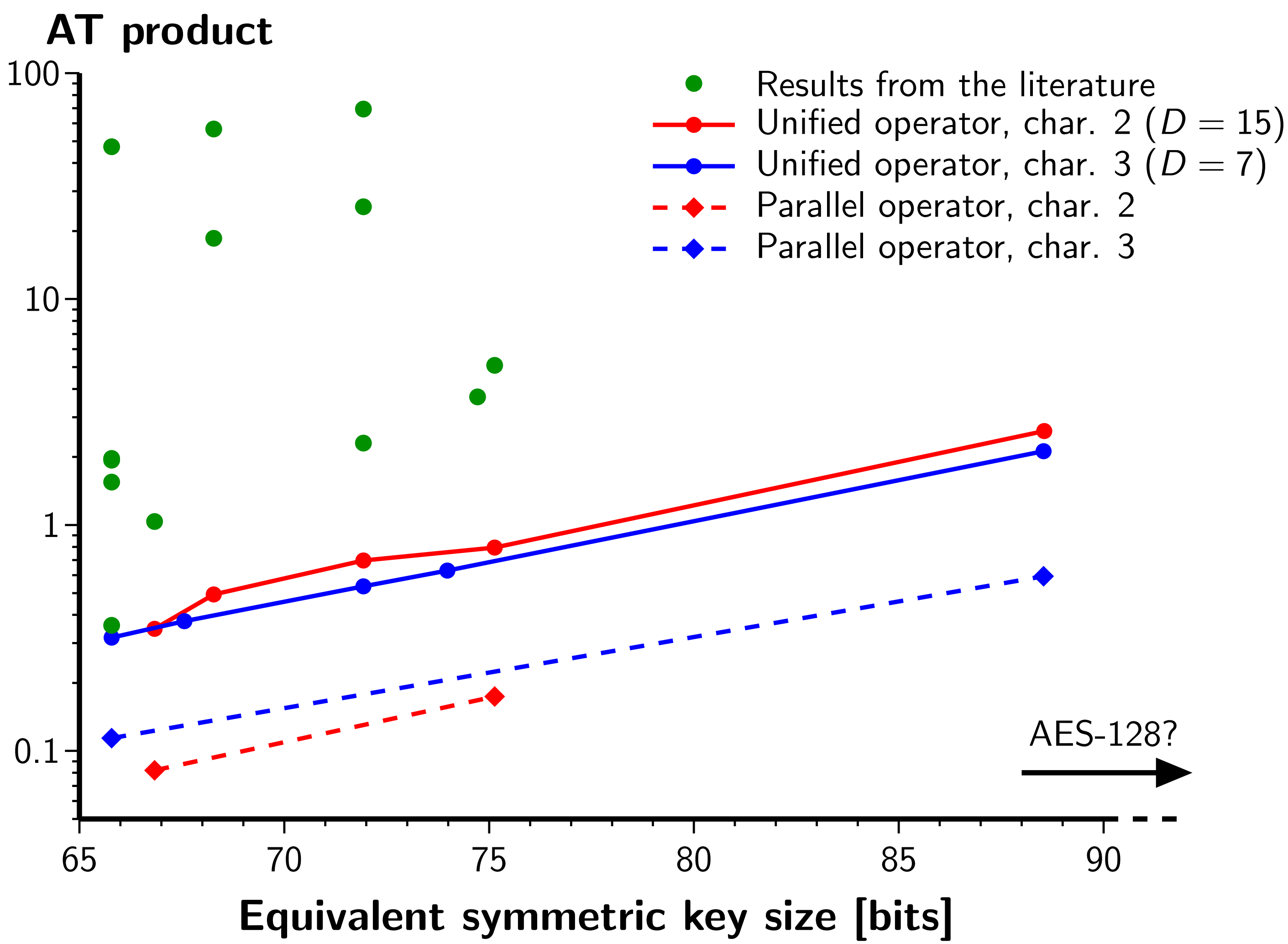
Area-time product



Area-time product



Area-time product



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- Scalability? **AES-128?**

With thanks to our sponsor

Thank you for your attention

Questions?